

Surds are numbers left in 'square root form' (or 'cube root form' etc). They are therefore irrational numbers. The reason we leave them as surds is because in decimal form they would go on forever.

Remember the following rules:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a} \times \sqrt{a} = a$$

ADDITION AND SUBTRACTION OF SURDS

Adding and subtracting surds is simple. Simply ensure that the roots of the numbers being added or subtracted are the same then simply add or subtract the non-root part.

$$4\sqrt{7} - 2\sqrt{7} = 2\sqrt{7}$$

$$5\sqrt{2} + 8\sqrt{2} = 13\sqrt{2}$$

MULTIPLICATION

$$\sqrt{5} \times \sqrt{15} = \sqrt{75}$$

$$\sqrt{25} \times \sqrt{3}$$

$$5\sqrt{3}$$

$$(1 + \sqrt{3})(2 - \sqrt{8})$$

$$2 - \sqrt{8} + 2\sqrt{3} - \sqrt{24}$$

$$2 - 2\sqrt{2} + 2\sqrt{3} - 2\sqrt{6}$$

DIVISION (*RATIONALISING THE DENOMINATOR*)

It is untidy to have a fraction that has a surd denominator. This can be 'tidied up' by multiplying the top and bottom of the fraction by the **conjugate** of the bottom surd. (*The conjugate of a surd is simply the same surd with its middle sign changed.*)

This is known as **rationalising the denominator**, since surds are irrational numbers and you are changing the denominator from an irrational to a rational number.

Example: Show that $\frac{\sqrt{5}-1}{3-\sqrt{5}} = \frac{1+\sqrt{5}}{2}$. In this case we must rationalise the denominator

by multiplying the top and bottom of the fraction by the conjugate of $3-\sqrt{5}$ which is $3+\sqrt{5}$.

$$\Rightarrow \frac{\sqrt{5}-1}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \Rightarrow \frac{3\sqrt{5}+5-3-\sqrt{5}}{9+3\sqrt{5}-3\sqrt{5}-5} \Rightarrow \frac{2+2\sqrt{5}}{4}$$

$$\Rightarrow \frac{2}{4} + \frac{2\sqrt{5}}{4} \Rightarrow \frac{1}{2} + \frac{\sqrt{5}}{2} \Rightarrow \frac{1+\sqrt{5}}{2}$$