

5TH YEAR HONOURS MATHEMATICS

SIMULTANEOUS LINEAR EQUATIONS IN THREE VARIABLES

The procedure for solving simultaneous linear equation in three variables is the very same used for solving simultaneous equations in two variables.

REVISION: In order to solve a simultaneous equation in two variables we paired off the two equations given, eliminated one of the variables by multiplication and subtraction if necessary, solved for the remaining variable and then used this value by substitution to find the value of the eliminated variable.

So, the steps were i) elimination, ii) solve iii) substitution iv) solve.

When we are asked to solve for three variables, three equations must be given. Then:

1. Make two pairs of equations, in any order, from the three equations given.
2. Eliminate the same letter from both sets of equations.
3. Use the two resulting equations to form a pair of simultaneous equations in two variables.
4. Eliminate another variable.
5. Solve for the remaining variable.
6. By substitution into earlier equations, solve for the other eliminated variables.

IMPORTANT POINTS TO REMEMBER:

- If there are fractions or brackets present in any of the given equations, they must be cleared before you place them in pairs.
- If any of the equations are missing one of the three variables, replace the missing variable back into the equation with 0 as its coefficient.

EXAMPLE: Solve for x, y and z.

$$x + y + z = 1$$

$$2x - 3y - 2z = -9$$

$$2x - 3z = -16$$

Since equation 3 is missing a 'y' term we can rewrite it as: $2x + 0y - 3z = -16$

This helps us to keep common terms together. Now forms the pairs:

$$x + y + z = 1$$

$$\underline{2x - 3y - 2z = -9}$$

$$x + y + z = 1$$

$$\underline{2x + 0y - 3z = -16}$$

Eliminate z from both equations to get:

$$4x - y = -7$$

$$\underline{5x + 3y = -13}$$

Now eliminate y from both equations:

$$17x = -34$$

$$\Rightarrow x = -2$$

Substitute for y into one of the above equations to find x:

$$4(-2) - y = -7$$

$$\Rightarrow y = -1$$

Now substitute these values into one of the original equations to find z:

$$-2 + (-1) + z = 1$$

$$\Rightarrow z = 4.$$