

A quadratic equation is one that contains a squared term.

The presence of this power of 2, reveals that the equation has two factors and hence two roots. There are three types of quadratic equation you may be asked to factorise and solve: (**Remember that you must factorise before you solve!**)

1. $ax^2 + bx + c = 0$, which is solved by the bracket method or by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ if asked to give the roots in surd form.

2. $ax^2 + bx = 0$ where common terms are extracted placed into a separate bracket and divided into the original equation e.g. $5x^2 - 10 = 0 \Rightarrow (5x)(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$.

3. $ax^2 - c = 0$, better known as the difference between two squares where each terms has two identical factors and they are placed into separate brackets e.g.

$$9x^2 - 16 = 0 \Rightarrow (3x + 4)(3x - 4) = 0 \Rightarrow x = -\frac{4}{3} \text{ or } \frac{4}{3}.$$

IF FRACTIONS OR BRACKETS ARE PRESENT IN THE ORIGINAL EQUATION, CLEAR THEM FIRST.

If the original equation contains a modulus or a square root, then all terms in this equation must be squared in order to form the quadratic equation.

The modulus is the absolute value of a term and is denoted by two bars $||$

e.g. $|4| = 4$ and $|-4| = 4$.

Since the term between the two bars may be positive or negative, square it and all terms to ensure a positive value. We now know it is positive after squaring so we may drop the modulus bars.

Gather similar terms together and solve the resulting quadratic equation.

Example: Solve $\left| \frac{3x+1}{x-1} \right| = 2$

To remove the modulus square both sides to get $\frac{9x^2 + 6x + 1}{x^2 - 2x + 1} = \frac{4}{1}$.

Cross multiply to get $9x^2 + 6x + 1 = 4x^2 - 8x + 4$

Gather similar terms and form the quadratic $5x^2 + 14x - 3 = 0$

Now solve by the bracket method $(5x - 1)(x + 3) = 0 \Rightarrow x = \frac{1}{5}$ or $x = -3$

The same procedure is followed when a $\sqrt{\quad}$ is present in the equation. To clear it you must square every term in the equation, gather similar terms together and solve the resulting quadratic equation.

Example: Solve $\sqrt{3x-2} = 2 + \sqrt{x-2}$

Square both sides to get $(\sqrt{3x-2})^2 = (2 + \sqrt{x-2})^2$

$\Rightarrow 3x - 2 = 4 + 4\sqrt{x-2} + x - 2$. Notice that the square still remains.

Rearrange the equation so that the squared term is on its own on the right hand side of the equation..

$\Rightarrow 3x - 2 - 4 - x + 2 = 4\sqrt{x-2} \Rightarrow 2x - 4 = 4\sqrt{x-2}$

Now square both sides again to clear the root.

$\Rightarrow 4x^2 - 16x + 16 = 16x - 32$

$\Rightarrow 4x^2 - 32x + 48 = 0$ (*all terms divisible by 4*) $\Rightarrow x^2 - 8x + 12 = 0$

Now solve by the bracket method to get $(x-6)(x-2) = 0 \Rightarrow x = 6$ or $x = 2$.