

# 5<sup>TH</sup> YEAR HONOURS MATHEMATICS

## NATURE OF THE ROOTS OF A QUADRATIC EQUATION

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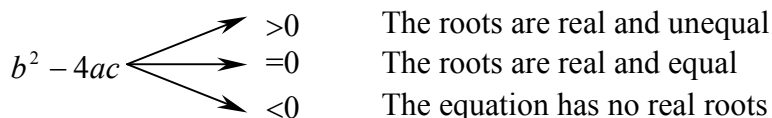
Take a quadratic equation  $ax^2 + bx + c = 0$ .

The quadratic formula  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  is used in order to reveal the roots of that or any quadratic equation where a is the coefficient of the squared variable, b the coefficient of the single variable and c is the constant term at the end of the equation.

When a, b and c are inserted, the formula reduces to two numbers as long as  $b^2 - 4ac$  remains  $\geq 0$ . If  $b^2 - 4ac$  gives a value of  $< 0$ , the equation cannot be solved in real terms, since we cannot get the square root of a negative number.

Therefore  $b^2 - 4ac$  determines the nature of the roots of a given quadratic.

There are three possibilities:



In order to see this in operation please use the following programme available from the school website: <http://www.knocklyoncs.ie/maths/roots.xls> Simply plug in a, b and c and see how these values cause the nature of the roots to change.

- If the quadratic has two real and unequal roots its graph cuts the x-axis twice in two different places.
- If the quadratic has two real and identical roots its graph is a tangent to the x-axis at that root.
- If the quadratic has no real roots its graph does not touch the x-axis at any point.

THEREFORE  $\Rightarrow$  THE ROOTS OF A QUADRATIC ARE REAL AS LONG AS  $b^2 - 4ac \geq 0$ .

**EXAMPLE:** Show that the roots of the equation  $x^2 - (2p + 5)x + 2(2p + 3) = 0$  are real for all values of  $p \in R$ .

**SOLUTION:** If the roots are real then  $b^2 - 4ac \geq 0$ .

Write down the values of a, b and c:  $a = 1$ ,  $b = -(2p + 5)$  and  $c = 2(2p + 3)$

It is important to remove brackets from b and c to get  $a = 1$ ,  $b = -2p - 5$  and  $c = 4p + 6$

To discover the nature of the roots of this quadratic we utilise  $b^2 - 4ac$ .

$$\begin{aligned} \Rightarrow (-2p - 5)^2 - 4(1)(4p + 6) &\Rightarrow 4p^2 + 20p + 25 - 16p - 24 &\Rightarrow 4p^2 + 4p + 1 \geq 0 \\ & &\Rightarrow (2p + 1)(2p + 1) \geq 0 \\ & &\Rightarrow (2p + 1)^2 \geq 0 \quad \checkmark \end{aligned}$$