5[TH] YEAR HONOURS MATHEMATICS

NATURE OF THE ROOTS OF A QUADRATIC EQUATION

Take a quadratic equation $ax^2 + bx + c = 0$.

The quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is used in order to reveal the roots of that or any quadratic equation where a is the coefficient of the squared variable, b the coefficient of the single variable and c is the constant term at the end of the equation.

When a, b and c are inserted, the formula reduces to two numbers as long as $b^2 - 4ac$ remains ≥ 0. If $b^2 - 4ac$ gives a value of < 0, the equation cannot be solved in real terms, since we cannot get the square root of a negative number.

Therefore $b^2 - 4ac$ determines the nature of the roots of a given quadratic.

There are three possibilities:

$b^2 - 4ac$	>0	The roots are real and unequal
	$=0$	The roots are real and equal
	<0	The equation has no real roots

In order to see this in operation please use the following programme available from the school website: http://www.knocklyoncs.ie/maths/roots.xls Simply plug in a, b and c and see how these values cause the nature of the roots to change.

- If the quadratic has two real and unequal roots its graph cuts the x-axis twice in two different places.
- If the quadratic has two real and identical roots its graph is a tangent to the x-axis at that root.
- If the quadratic has no real roots its graph does not touch the x-axis at any point.

THEREFORE ⇒ THE ROOTS OF A QUADRATIC ARE REAL AS LONG AS $b^2 - 4ac \geq 0$.

EXAMPLE: Show that the roots of the equation $x^2 - (2p+5)x + 2(2p+3) = 0$ are real for all values of $p \in R$.

SOLUTION: If the roots are real then $b^2 - 4ac \geq 0$.

Write down the values of a, b and c: $a = 1$, $b = -(2p+5)$ and $c = 2(2p+3)$

It is important to remove brackets from b and c to get $a = 1$, $b = -2p - 5$ and $c = 4p + 6$

To discover the nature of the roots of this quadratic we utilise $b^2 - 4ac$.

$\Rightarrow (-2p-5)^2 - 4(1)(4p+6) \Rightarrow 4p^2 + 20p + 25 - 16p - 24 \Rightarrow 4p^2 + 4p + 1 \geq 0$

$\Rightarrow (2p+1)(2p+1) \geq 0$

$\Rightarrow (2p+1)^2 \geq 0$ ✓