LOGARITHMS

In mathematics, a **logarithm,** or **log** for short, of a given number to a given base is the power to which you need to raise the base in order to get the number.
For example, the logarithm of 1000 to the common base 10 is 3, because 10 raised to a power of 3 is 1000. So in short a log is a power!

Take, for example, $a = b^c$. Since c is the power, c is the log! Since b is the base number in $a = b^c$, b will also be the base of the log. Therefore we can rewrite the equation $a = b^c$ in log form $\Rightarrow \log_b a = c$. The base of the log and the base of its original index form are always the same e.g. $\log_2 8 = 3$ can be rewritten as $2^3 = 8$. It is vital for this aspect of your course that you know how to interchange between index form and log form.

An easy way to solve simple logs is to read them diagonally! The answer to $\log_2 32$ is the power that 2 must be raised to in order to give 32. So if we read the log diagonally from the base number we see that $2^5 = 32$, so the answer to this log is 5.

RULES OF LOGS

$\log_b(mn) \Leftrightarrow \log_b(m) + \log_b(n)$
$\log_b(m/n) \Leftrightarrow \log_b(m) - \log_b(n)$
$\log_b(m^n) \Leftrightarrow n \cdot \log_b(m)$
$\log_n m \Leftrightarrow \dfrac{\log_b m}{\log_b n}$
$\log_b b \Leftrightarrow 1$
$\log_b 1 \Leftrightarrow 0$
$\log x \Leftrightarrow \log_{10} x$
$\ln x \Leftrightarrow \log_e x$

The $\Leftrightarrow$ symbol shows that the rules can be used backwards as well as forwards.

The last two logs deserve special mention.

- The first is $\log_{10} x$, known as a common log. This is the only log that can be written without its base e.g. log1000 = 3. The log button on your calculator works to base 10.
- The second is $\ln x$ or $\log_e x$, known as a natural log. *e* is an irrational number just like π. Its value is 2.71828183….. When natural logs or *e* are used in a question, they must be cleared. It is good to remember that $e^{\ln x}$ and $\ln e^x$ both give *x* as the answer. These two results are used to clear the *ln* or the *e* if they should be present in any question:

  EXAMPLE: Solve $e^{2x} = 3$

  $\Rightarrow$ take the ln of both sides $\Rightarrow \ln e^{2x} = \ln 3 \Rightarrow 2x = \ln 3 \Rightarrow x = \dfrac{1}{2}\ln 3$

The golden rule with working with logs is to ensure that all logs in any given question have the same base number. If not, your first job will be to use the 'change of base' rule to make them all the same.

When equations have two or more logs to the same base, sometimes we utilise substitution of a letter for the log in order to simplify the equation:

EXAMPLE: Solve $4\log_x 2 = \log_2 x + 3$

$\Rightarrow 4\left(\dfrac{\log_2 2}{\log_2 x}\right) = \log_2 x + 3 \Rightarrow 4\left(\dfrac{1}{\log_2 x}\right) = \log_2 x + 3$. Now let $\log_2 x = y$ and substitute:

$\Rightarrow 4\left(\dfrac{1}{y}\right) = y + 3$. This equations is now multiplied by 'y' to clear the fraction

$\Rightarrow 4 = y^2 + 3y \Rightarrow y^2 + 3y - 4 = 0 \Rightarrow (y+4)(y-1) = 0 \Rightarrow y = -4$ and $y = 1$.

Now undo the substitution by replacing $\log_2 x = y$

$\Rightarrow \log_2 x = -4$ and $\log_2 x = 1$

$\Rightarrow x = \dfrac{1}{16}$ and $x = 2$

USING A CALCULATOR TO WORK OUT LOGS

In a question like the following where we are asked to solve for an unknown in an index, we make use of common logs and then of the calculator!

Solve for n in $5^{n-1} = 2^n$.

Get the $\log_{10}$ of both sides $\Rightarrow \log 5^{n-1} = \log 2^n \Rightarrow n-1.\log 5 = n.\log 2$

Now work out log 5 and log 2 with the calculator: $\Rightarrow n - 1(0.7) = n(0.3)$

$\Rightarrow 0.7n - 0.7 = 0.3n$

$\Rightarrow 0.7n - 0.3n = 0.7$

$\Rightarrow 0.4n = 0.7$

$\Rightarrow n = 1.75$