

5TH YEAR HONOURS MATHEMATICS

SIMULTANEOUS EQUATIONS, ONE LINEAR AND ONE QUADRATIC

When we are given a pair of simultaneous equations to solve where one is linear and the other is quadratic, we solve for the variables by substitution. If the equation has an x^2 and a y^2 then there will be two answers for both x and y .

METHOD:

Begin with the linear equation. Manipulate it so that you express one variable in terms of the others. Now substitute this value into the quadratic equation in the appropriate places. Solve the quadratic that forms to find one variable and by substitution find two values for the second variable by substitution of these values into the original linear equation.

Example: Solve the following pair of simultaneous equations:

$$\begin{aligned} 2x + 3y + 4 &= 0 \\ (x + 3y)(2x - y) &= 4 \end{aligned} \quad \Rightarrow \text{Manipulate equation 1 to get } 2x = -3y - 4 \Rightarrow x = \frac{-3y - 4}{2}$$

Equation 2 must be expanded to remove the brackets:

$$2x^2 - xy + 6xy - 3y^2 = 4 \Rightarrow 2x^2 + 5xy - 3y^2 = 4$$

Now substitute the result of equation 1 into equation 2.

$$\Rightarrow 2\left(\frac{-3y - 4}{2}\right)^2 + 5\left(\frac{-3y - 4}{2}\right)y - 3y^2 = 4 \Rightarrow 2\left(\frac{9y^2 + 24y + 16}{4}\right) + 5\left(\frac{-3y - 4}{2}\right)y - 3y^2 = 4$$

Multiply every term by 2 to clear all fractions:

$$\begin{aligned} \Rightarrow 9y^2 + 24y + 16 + 5(-3y - 4)y - 6y^2 &= 8 \Rightarrow 9y^2 + 24y + 16 - 15y^2 - 20y - 6y^2 = 8 \\ \Rightarrow -12y^2 + 4y + 8 &= 0 \Rightarrow 3y^2 - y - 2 = 0 \end{aligned}$$

Now factorise by the bracket method $\Rightarrow (3y + 2)(y - 1) = 0 \Rightarrow y = -\frac{2}{3}$ or $y = 1$

We solve for x by substitution of the two y values into the original linear equation:

$$\Rightarrow 2x + 3\left(-\frac{2}{3}\right) + 4 = 0 \Rightarrow 2x + 2 = 0 \Rightarrow x = -1$$

Or

$$\Rightarrow 2x + 3(1) + 4 = 0 \Rightarrow 2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$$

SOLUTION: $x = -1, y = -\frac{2}{3}$ or $x = -\frac{7}{2}, y = 1$
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