AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE

MATHEMATICS SYLLABUS

FOUNDATION LEVEL

LEAVING CERTIFICATE

MATHEMATICS SYLLABUS

Foundation Level

1 Introduction

Context

Mathematics is a wide-ranging subject with many aspects. On the one hand, in its manifestations in terms of counting, measurement, pattern and geometry it permeates the natural and constructed world about us, and provides the basic language and techniques for handling many aspects of everyday and scientific life. On the other hand, it deals with abstractions, logical arguments, and fundamental ideas of truth and beauty, and so is an intellectual discipline and a source of aesthetic satisfaction. These features have caused it to be given names such as the "queen and the servant of the sciences". Its role in education reflects this dual nature: It is both practical and theoretical, geared to applications and of intrinsic interest, with the two elements firmly interlinked.

Mathematics has traditionally formed a substantial part of the education of young people in Ireland throughout their schooldays. Its value for further and higher education, for employment, and as a component of general education has been recognised by the community at large. Accordingly, it is of particular importance that the mathematical education offered to students should be appropriate to their abilities, needs and interests, and should fully and appositely reflect the broad nature of the subject and its potential for enhancing the students' development.

1.2 AIMS

It is intended that mathematics education would:

(a) Contribute to the personal development of the students:

- helping them to acquire the mathematical knowledge, skills and understanding necessary for personal fulfilment;
- developing their modelling abilities, problem-solving skills, creative talents, and powers of communication;
- extending their ability to handle abstractions and generalisations, to recognise and present logical arguments, and to deal with different mathematical systems;
- fostering their appreciation of the creative and aesthetic aspects of mathematics, and their recognition and enjoyment of mathematics in the world around them;
- hence, enabling them to develop a positive attitude towards mathematics as an interesting and valuable subject of study;

(b) Help to provide them with the mathematical knowledge, skills and understanding needed for life and work:

- promoting their confidence and competence in using the mathematical knowledge and skills required for everyday life, work and leisure;
- equipping them for the study of other subjects in school;
- preparing them for further education and vocational training;
- in particular, providing a basis for the further study of mathematics itself.

It should be noted that in catering for the needs of the students, the courses should also be producing suitably educated and skilled young people for the requirements of the country.

1.3 Objectives

The teaching and learning of mathematics has been described as involving facts, skills, concepts (or "conceptual structures"), strategies, and – stemming from these – appreciation.

In terms of student outcomes, this can be formulated as follows. The students should be able to recall relevant facts. They should be able to demonstrate instrumental understanding ("knowing how") and necessary psychomotor skills. They should possess relationship understanding ("knowing why"). They should be able to apply their knowledge in familiar and eventually in unfamiliar contexts; and they should develop analytical and creative powers in mathematics. Hence, they should develop appreciative attitudes to the subject and its uses. The aims listed in Section 1.2 can therefore be translated into general objectives as given below.

(a) Students should be able to recall basic facts; that is, they should be able to:

- display knowledge of conventions such as terminology and notation;
- recognize basic geometrical figures and graphical displays;
- state important derived facts resulting from their studies.

(Thus, they should have fundamental information readily available, to enhance understanding and aid application).

(b) They should be able to demonstrate instrumental understanding; hence they should know how (and when) to:

- carry out routine computational procedures and other such algorithms;

- perform measurements and constructions to an appropriate degree of accuracy;

- present information appropriately in tabular, graphical and pictorial form, and read information presented in these forms;

- use mathematical equipment such as calculators, rulers, setsquares, protractors, and compasses, as required for the above.

(Thus, they should be equipped with the basic competencies needed for mathematical activities).

© They should have acquired relational understanding, i.e. understanding of concepts and conceptual structures, so that they can:

- interpret mathematical statements;
- interpret information, presented in tabular, graphical and pictorial form;
- recognize patterns, relationships and structures;
- follow mathematical reasoning.

(Thus, they should be able to see mathematics as an integrated, meaningful and logical discipline).

(d) They should be able to apply their knowledge of facts and skills; that is, they should be able when working in familiar types of context to:

- translate information presented verbally into mathematical form;
- select and use appropriate mathematical formulae or techniques in order to process the information;
- draw relevant conclusions.

(Thus, they should be able to use mathematics and recognize it as a powerful tool with wide ranging areas of applicability).

(e) They should have developed the psychomotor and communicative skills necessary for the above.

(f) They should appreciate mathematics as a result of being able to:

- use mathematical methods successfully;
- acknowledge the beauty of form, structure and pattern;
- recognize mathematics in their environment;
- apply mathematics successfully to common experience.

(g) They should be able to analyse information, including information presented in unfamiliar contexts:
- formulate proofs,
- form suitable mathematical models;
- hence select appropriate strategies leading to the solution of problems.

(h) They should be able to create mathematics for themselves:
- explore patterns;
- formulate conjectures;
- support, communicate and explain findings.

(i) They should be aware of the history of mathematics and hence of its past, present and future role as part of our culture

<u>Note</u>

Many attempts have been made to adapt the familiar Bloom taxonomy to suit mathematics education: in particular, to include a category corresponding to "carrying out routine procedures". The categories used above are intended, inter alia, to facilitate the design of suitably structured examination questions.

2. Foundation Level

2.1 Rationale

The Foundation Course is intended to equip students with the knowledge and techniques required in everyday life and in various kinds of employment. It is also intended to lay the groundwork for students who proceed to further education and training in areas in which specialist mathematics is not required. It should therefore provide students with the mathematical tools needed in their daily life and work and (where relevant) continuing study; but it should do so in a context designed to build the students' confidence, their understanding and enjoyment of mathematics, and their recognition of its role in the world around them. Hence, material is chosen for its intrinsic interest and immediate applicability as well as its usefulness beyond school.

The course is designed for students who have had only very limited acquaintance with abstract mathematics. Basic knowledge is maintained and enhanced by being approached in an exploratory and reflective manner –available of students' increasing maturity – rather than by simply repeating work done in the Junior Cycle. Concreteness is provided in particular by extensive use of the calculator; this serves as an investigative tool as well as an object of study and a readily available resource. By means of such a developmental and constructive approach, the ground is prepared for students' advance to abstract concepts via a multiplicity of carefully graded examples. Computational work is balanced by emphasis on the visual and spatial.

For the target group, particular emphasis can be given to aims concerned with the use of mathematics in everyday life and work – especially as regards intelligent and proficient use of calculators – and with the recognition of mathematics in the environment.

2.2 AIMS

In the light of the aims of mathematics education listed in Section 1.2, the aims of the Foundation course are:

- development of students' understanding of mathematical knowledge and techniques required in everyday life and employment;
- particular emphasis of meaningfulness of mathematical concepts;
- acquisition of mathematical knowledge that is of immediate applicability and usefulness.
- introduction of the students to mathematical abstraction;
- maintenance and enhancement of students' basic mathematical knowledge and skills;
- encouragement of accurate and efficient use of the calculator;
- promotion of students' confidence in working with mathematics.

2.3 Assessment Objectives

The assessment objectives are the objectives (a), (b), (c), (d) and (e) listed in Section 1.3. These objectives should be interpreted in the context of the statement of the aims of the Foundation course. Knowledge of the content of the Junior Certificate Foundation course will be assumed.

2.4 Structure and Content

The syllabus is presented without options. It is therefore envisaged that students would study the entire course.

CONTENT

Number Systems

Revision of the following, using calculator for all relevant aspects:

1. Development of the systems N of natural numbers, Z of integers,
   Q of rational numbers and R of real numbers.
   The operations of addition, multiplication, substraction and division.
   Representation of numbers on a line.
   inequalities.
   Decimals
   Powers and roots.
   Scientific notation.
2. Factors, multiples, prime numbers in N. Prime factorization
3. Use of brackets. Conventions as to the order of precedence of operations.

Arithmetic.

Use of calculator for all relevant operations in the following:

1.	Approximation and error; rounding off. Relative error, percentage error, tolerance. Very large and very small numbers on the calculator. Limits to accuracy of calculators.	
2.	Substitution in formulae.	Main stages of calculation should be shown.
3.	Proportion, Percentage, Averages. Average rates of change (with respect to time).	
4.	Compound interest and depreciation formulae.	Formula provided in examinations; **n** a natural number.

$$A = P\left\{1 + \frac{r}{100}\right\}^{n}$$

$$P = A/\left\{1 + \frac{r}{100}\right\}^{n}$$

5. Value Added Tax (VAT), Rates. Income Tax (including PRSI); emergency tax; tax tables.
6. Domestic bills and charges.
7. Currency transactions, including commission.
8. Costing. Materials and labour. Wastage.
9. Metric system. Change of units. Everday imperial units. — Conversion factors provided for imperial units.

Areas and Volumes

Use of calculator for all relevant operations in the following:

1. Plane figures: disc, triangle, rectangle, square, H-figure, parallelogram, trapezium.
   Solid figures: right cone, rectangular block, cylinder, sphere, right prism. — See Appendix Questions will be confined to the variables Given in the formulae (" engineer's handbook" approach).
2. Use of Simpson's Rule to approximate area.

Algebra

1. Consideration of the following, using calculator where relevant:

(j)	$x + a = b;$ }		
(ii)	$ax = b;$ }	$a,b,c,\varepsilon\ Q$	
(iii)	$ax + b = c;$ }		
(iv)	$ax + b = cx;$	$a, b, c, \varepsilon\ Z$	
(v)	$ax + b = cx + d;$	$a, b, c, d, \varepsilon\ Z$	
(vi)	$ax + by = c;$ }	$a, b, c, d, e,$	Cases with unique solutions only.
	$dx + ey = f;$ }	$f\ \varepsilon\ Z$	

Problems giving rise to equations of type (i) - (vi)

(vii)	$x^2 = a;$	$a\ \varepsilon\ Q+$		
viii)	$x^2 + a = b;$	$b - a > 0_1$	$a, b\ \varepsilon\ Q$	
ix)	$ax^2 = b;$	$a, b\ \varepsilon\ Q^+$		
x)	$ax^2 + b = c;$	$a > 0, (c-b) > 0$	$a, b, c\ \varepsilon\ Z$	
xi)	$ax^2 + bx + c = 0;$	$a > 0, b^2 \geq 4ac,$	$a, b, c\ \varepsilon\ Z$	Use of formula (provided in Examinations

2. Consideration of the inequalities:

i)	$x + a > b;$	$x + a <b;$ }	
ii)	$ax > b;$	$ax < b;$ }	
iii)	$ax + b > c;$	$ax + b < c)$ }	$a, b, c\ \varepsilon\ Z$
iv)	$x + ab \geq b;$	$x + a \leq b;$ }	
v)	$ax \geq b;$	$ax \leq b;$ }	
vi)	$ax + b \geq c;$	$ax + b \leq c;$ }	

Statistics and probability

1.	Fundamental Principle of Counting: if one task can be accomplished in **x** different ways, and following this a second task can be accomplished in y different ways, then the first task followed by the second task can be accomplished in **xy** different ways.	Use in examples
2.	Discrete probability; simple cases. For equally likely outcomes, probability = (number of outcomes of interest)/ (number of possible outcomes)	Examples including coin tossing, dice throwing, birthday distribution, card drawing (one or two cards) and sex distribution
3.	Statistics: graphical and tabular representation of statistical data; grouped and ungrouped frequency distributions. Mean: cumulative frequencies and cumulative frequency graph; median; weighted mean. Concept of dispersion; standard deviation of ungrouped Array of not more than ten numbers	Emphasis on use of calculator Median obtained from array or cumulative frequency graph: finding median from histogram excluded (but histogram itself included)

Trignometry

1.	Sine, cosine and tangent at ratios in a right-angled triangle	
2.	Solving for an unknown in a right-angled triangle	Problems to include diagrams.

Functions and graphs

1	A function as a set of couples	A function as a special relation, hence a particular form of association between the elements of two sets.

Function considered as specified by a formula or rule	Establishment of such an association

Use of notation

f (x) =
f; x →
y =

2. Study of the following functions and of equations of the Form f(x) = k, k ε Z:

f: x → mx;	m ε Q,	x ε R	Effect on the graph of varying m
f: x → mx + c;	m, c ε Q,	x ε R	Significance of c
f: x → x^2;	x ε R		For example, estimation of √2
f: x → x^2 + c;	c, x ε R		Effect of the graph of varying c
f: x → ax^2 ;	a, x ε R		Effect of the graph of varying a

f: x → ax^2 + bx + c; a, b, c, x ε R Values of x for which f(x) is maximum/minimum. Intervals of x for which f(x) is increasing/decreasing.

3. Experimental results. Fitting a straight line to a set of experimental data. Prediction.

4. Interpretation of graphs in following cases:

Case 1	**Examples**
Cases in which information is available only at plotted points	- currency fluctations - inflation - employment/unemployment - temperature - temperature chart (medical) - pollen count

- lead levels
- smog

Case 2

Continuous graphs - distance/time - speed/time - depth of liquid/time - conversion of units	Interpretation to include: given range of values of one variable, estimate from the graph the corresponding range of values of the other

Geometry

1.	Co-ordinate geometry: Distance between two points Slope of a line through two points. Parallel lines Perpendicular lines.	Formulae will be given in examinations
	Midpoint of a line segment	
	Equation of line: $y = mx + c$	Obtaining equation of line, given slope and one point or given two points
2.	Geometrical results – knowledge of the following and use in numerical examples:	Proofs excluded
	(a) Vertical opposite angles are equal:	"Equal" means equal in measure.
	(b) When a transversal cuts two parallel lines The corresponding angles are equal, and the Alternate angles are equal.	

© Opposite sides and angles of a parallelogram are equal

(d) The sum of the angles of a triangle is 180^o;

(e) The base angles of an isosceles triangle are equal

(f) The angle on a (straight) line is 180^o;

(g) The Theorem of Pythagoras;

(h) The angle in a semicircle is a right angle.

3. Construction

(a) To draw a perpendicular from a given point on a line

(b) To draw a perpendicular from a point at the end of a line segment;

(c) To draw a perpendicular to a given line from a point not on the line

(d) To construct an angle of 60^o;

(e) To construct an angle equal to a given angle;

(f) To draw a line parallel to a given line through a point;

(g) To construct a parallelogram (given sufficient data);

(h) To draw the circumscribed circle of a given triangle;

(i) To draw the inscribed circle of a given triangle

(j) To draw the tangent to a circle at a given point on the circle.

4. Enlargements

main emphasis on construction

Enlargement of a rectilinear figure by the ray method. Centre of enlargement. Scale factor k. Two cases to be Considered.

- $k>1$, $k \varepsilon Q$ (enlargement)
- $O < k < 1$, $k \varepsilon O$ (reduction).

A triangle abc with centre of enlargement a, enlarged by a scale factor k, gives an image triangle ab'c' with bc parallel to b'c'

Object length, image length, calculation of scale factor.
Finding the centre of enlargement.
A rectilinear region when enlarged by a scale factor k has its area multiplied by a factor k^2.

5. Repeating patterns. Identification of axial symmetry. planes in symmetry, central symmetry, and rotational symmetry in given figures. Patterns in different cultures. Co-ordinate treatment not included.

6 Assessment

It is envisaged that, at present, the courses would be assessed by means of final written examinations.

The following principles would apply:

(a) The status and standing of the Leaving Certificate would be maintained
(b) Candidates would be able to demonstrate what they know rather than what they do not know
(c) Examinations would build candidates; confidence that they can do mathematics, rather than undermining the confidence of those who attempt them.

Note

Restriction at present to assessment by formal written examination has governed the specification of assessment objectives (for those for the Foundation Course, see Section 2.3); they have been limited to a subset of the general objectives (see Section 1.3). In the future, it may be possible to introduce a coursework component. This would facilitate assessment of the other objectives, notably problem-solving, communicative and creative skills, and in particular of work done with the aid of computers.

Select Bibliography

Among the national and international literature consulted, the following three national reports are of particular relevance:

1. Curriculum and Examination Board: Mathematics Education: Primary and Junior Cycle Post-Primary. Dublin: Curriculum and Examinations Board 1986.

2. Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr. W. H. Cockcroft. (The Cockcroft Report). London: HMSO, 1982.

3. National Research Council: Everybody Counts: a Report to the Nation on the Future of Mathematics Education: Washington, D.C: National Academy Press 1989.

Appendix "Engineer's Handbook"

The handbook is intended to be used as follows:

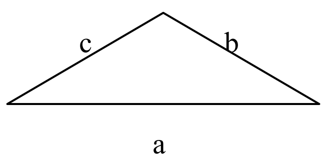
- students select the formula with the required unknown on the left-hand side and with values available for all variables on the right-hand side

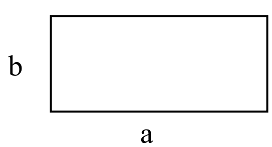
- they substitute values for variables on the right-hand side of the formula;

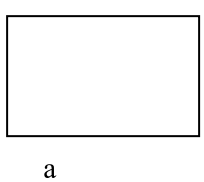
- they evaluate the required answer, typically using a calculator.

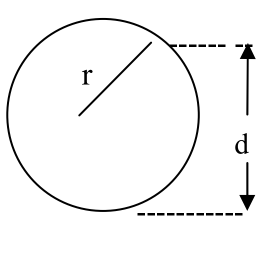
This obviates algebraic manipulation. It also provides experience of an approach widely used in practical applications.

Length

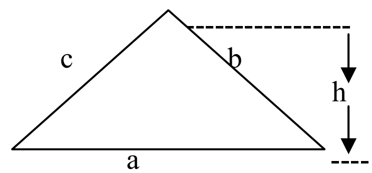
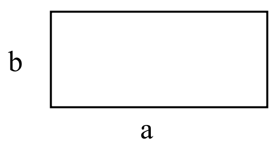
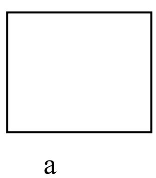
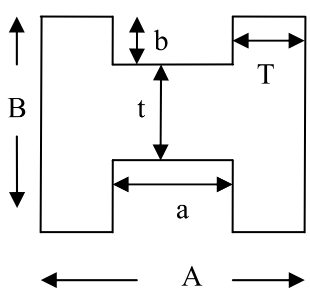
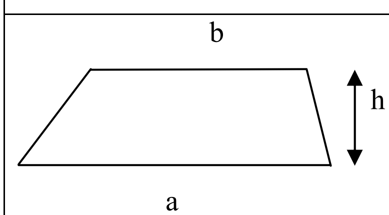
TRIANGLE	LENGTH (L)	FORMULAE
	$L = a+b+c$	$a = L- b-c$ $b = L-a-c$ $c= L-a -b$

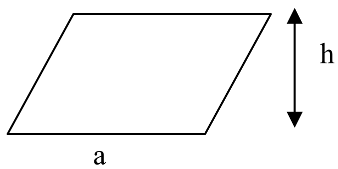
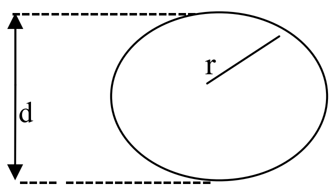
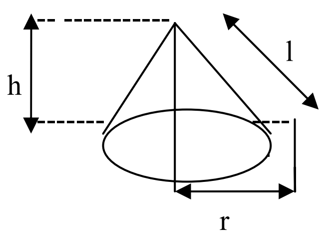
RECTANGLE	LENGTH (L)	FORMULAE
	$L = 2\ (a+ b) = 2a + 2b$	$a = \frac{(L - 2b)}{2}$ $b = \frac{(L - 2a)}{2}$

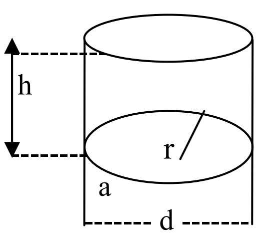
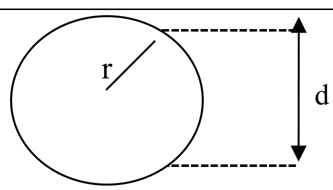
SQUARE	LENGTH (L)	FORMULAE
	$L = 4a$	$A = \frac{L}{4}$

CIRCLE	LENGTH (L)	FORMULAE
	$L = 2\pi$ $L = \pi d$	$d = 2r \quad r = \frac{d}{2}$ $r = \frac{L}{2\ \pi}$ $d = \frac{L}{\pi}$

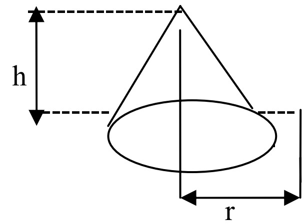
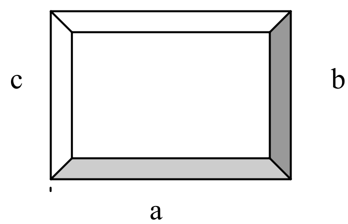
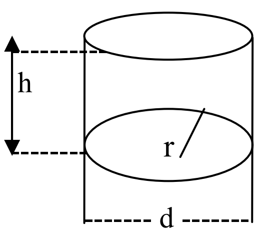
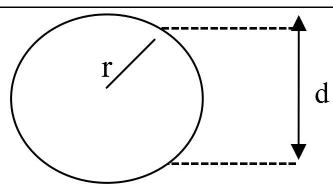
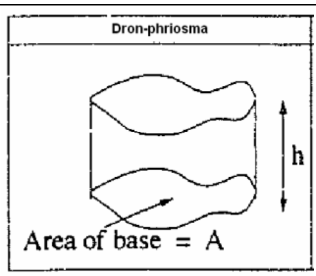
AREA

TRIANGLE	AREA	FORMULAE
	Area = $\frac{ab}{2}$	$a = \frac{2\,(area)}{h}$ $h = \frac{2\,(area)}{a}$
RECTANGLE	**AREA**	**FORMULAE**
	Area = ab	$a = \frac{Area}{b}$ $b = \frac{Area}{a}$
SQUARE	**AREA**	**FORMULAE**
	Area = a^2	A = $\sqrt{Area}$
H FIGURE	**AREA**	**FORMULAE**
	Area = AB – 2ab Area = at + 2 BT Note: A = a + 2 T B = 2b + t	$A = \frac{(Area + 2ab)}{B}$ $B = \frac{(Area + 2ab)}{A}$ $a = \frac{(AB - Area)}{2b}$ $b = \frac{(AB - Area)}{2a}$
TRAPEZIUM	**AREA**	**FORMULAE**
	Area = $\frac{h(a+b)}{2}$	$a = \frac{2\,(Area)}{h} - b$ $b = \frac{2\,(Area)}{h} - a$ $h = \frac{2\,(Area)}{(a + b)}$

PARALLELOGRAM	AREA	FORMULAE
	Area = ah	$a = \frac{\text{Area}}{H}$ $h = \frac{\text{Area}}{a}$
DISC	**AREA**	**FORMULAE**
	$\text{Area} = \pi r^2$ $\text{Area} = \frac{\pi d^2}{4}$	$R = \sqrt{\frac{\text{Area}}{\pi}}$ $d = \sqrt{\frac{4\ (\text{Area})}{\pi}}$
RIGHT CONE	**AREA**	**FORMULAE**
	$\text{Area} = \pi r l$ Note: $l^2 = r^2 + h^2$	$R = \frac{\text{Area}}{\pi l}$ $l = \frac{\text{Area}}{\pi r}$

CYLINDER	AREA	FORMULAE
	$\text{Area} = 2 \pi r h$ $\text{Area}\ \ \pi d h$	$r = \frac{\text{Area}}{2 \pi h}$ $h = \frac{\text{Area}}{2 \pi r}$ $d = \frac{\text{Area}}{\pi h}$ $h = \frac{\text{Area}}{\pi d}$
SPHERE	**AREA**	**FORMULAE**
	$\text{Area} = 4 \pi r^2$ $\text{Area} = \pi d^2$	$r = \sqrt{\frac{\text{Area}}{4 \pi}}$ $d = \sqrt{\frac{\text{Area)}}{\pi}}$

VOLUME

RIGHT CONE	**VOLUME (V)**	**FORMULAE**
	$V = \frac{\pi r^2 h}{3}$	$r = \sqrt{\frac{3V}{\pi h}}$ $h = \frac{3V}{\pi r^2}$
RECTANGULAR BLOCK	**VOLUME (V)**	**FORMULAE**
	$V = abc$	$a = \frac{V}{bc}$ $b = \frac{V}{ac}$ $c = \frac{V}{ab}$
CYLINDER	**VOLUME (V)**	**FORMULAE**
	$V = \pi r^2 h$ $V = \frac{\pi h d^2}{4}$	$h = \frac{V}{\pi r^2}$ $h = \frac{4V}{\pi d^2}$ $r = \sqrt{\frac{V}{\pi h}}$ $d = \sqrt{4\frac{V}{\pi h}}$
SPHERE	**VOLUME (V)**	**FORMULAE**
	$V = \frac{4\pi r^3}{3}$ $V = \frac{\pi d^3}{6}$	Cube roots required
RIGHT PRISM	**VOLUME (V)**	**FORMULAE**
	$V = Ah$	$A = \frac{V}{h}$ $h = \frac{V}{A}$