Looking Ahead to Chapter 14

Focus In Chapter 14, you will learn about different number systems, working with rational numbers, irrational numbers, and real numbers. You will also learn about the properties of number systems, including the distributive property.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 14.

Perform the indicated operation.

1. $\frac{3}{4} + 2\frac{5}{8}$

2. $\frac{5}{9} \times 1\frac{3}{4}$

3. $\frac{6}{7} \div \frac{9}{2}$

Evaluate the power.

4. 9^2

5. 3^4

6. 5^3

Write each number as a power with a negative exponent.

7. $\frac{1}{3^2}$

8. $\frac{1}{5^4}$

9. $\frac{1}{2^4}$

Solve the equation. Write your answer as a decimal.

10. $4x - 12 = 45$

11. $75 - 6y = 36$

12. $14z + (4 - 38) = 85$

13. Write the number that is the additive inverse of –8.

14. Write the number that is the multiplicative inverse of $\frac{4}{3}$.

Write an example of each property.

15. Associative Property of Multiplication

16. Commutative Property of Multiplication

Key Terms

natural number ● p. 462
whole number ● p. 462
integer ● p. 463
rational number ● p. 465
power ● p. 467
terminating decimal ● p. 472
repeating decimal ● p. 472
bar notation ● p. 472
irrational number ● p. 474
real number ● p. 475
Venn diagram ● p. 475
distributive property ● p. 481

14

Number Systems

Since 1983, cellular telephones have changed from being expensive pieces of equipment used by businesspeople to low-cost items used by people of all ages, including children. In Lesson 14.2, you use powers of rational numbers to determine the number of people in the U.S. who own cellular phones.

14.1 Is It a Bird or a Plane?

Rational Numbers

Objectives

In this lesson, you will:

- Use a number line to compare and order rational numbers.
- Learn about types of numbers and their properties.
- Perform operations with rational numbers.

Key Terms

- natural number
- whole number
- integer
- rational number

Take Note

Because paper is typically sold in 500-sheet quantities, a paper's weight is determined by the weight of 500 sheets of the paper. So, 500 sheets of 20-pound paper weighs 20 pounds.

In this lesson, we will explore several sets of numbers and their properties.

Problem 1 A Science Experiment

Your science class is conducting an experiment to see how the weight of a paper airplane affects the distance that it can fly. Your class is divided into two groups. Group 1 uses a yard stick to measure the distances that an airplane flies and Group 2 uses a meter stick. Group 2 then takes their measurements in meters and converts them to feet. The results of the experiment are shown in the table.

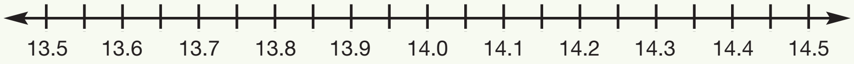
Type of Paper	Group 1 Measurements	Group 2 Converted Measurements
20-pound paper	$13\frac{7}{8}$ feet	13.9 feet
28-pound paper	$14\frac{3}{8}$ feet	14.4 feet

A. For each type of paper, your science class needs to compare the Group 1 measurement to the Group 2 converted measurement. In Lesson 4.6, we learned how to write a fraction as a decimal.

Write $13\frac{7}{8}$ as a decimal.

Write $14\frac{3}{8}$ as a decimal.

B. On the number line below, graph the Group 1 measurements written as decimals and the Group 2 converted measurements.



Investigate Problem 1

1. Use the number line to determine which group's plane traveled farther for the 20-pound paper and for the 28-pound paper. Write your answers using complete sentences.

Problem 2

Natural Numbers, Whole Numbers, and Integers

Fractions and decimals belong to a special set of numbers that you will learn about in Problem 3. This set includes other numbers that you already know—natural numbers, whole numbers, and integers.

A. The first set of numbers that you learned when you were very young was the set of counting numbers, or **natural numbers.** This set of numbers consists of the numbers that we use to count objects: 1, 2, 3, 4, 5, … .

How many counting numbers are there?

Does it make sense to ask which counting number is the largest? Use a complete sentence to explain why or why not.

Why do you think this set of numbers is called the natural numbers? Write your answer using a complete sentence.

Take Note

In the list

1, 2, 3, 4, 5, …

the dots at the end of the list mean that the list of numbers goes on without end.

B. You have used the set of **whole numbers** since grade school. This set is made up of the set of natural numbers and the number 0, the additive identity. Why is zero the additive identity? Write your answer using a complete sentence.

Other than being used as the additive identity, how else is zero used in the set of whole numbers? Write your answer using a complete sentence.

Use a complete sentence to explain why having zero makes the set of whole numbers more useful than the set of natural numbers.

Problem 2 Natural Numbers, Whole Numbers, and Integers

C. In Chapter 7, you used another set of numbers, the *integers*. This set includes all of the whole numbers and their additive inverses.

What is the additive inverse of a number? Write a complete sentence to explain your answer.

We wrote 1, 2, 3, 4, 5, … to represent the set of natural numbers. Represent the set of integers. Remember to use three dots to show that the numbers go on without end in both directions.

Does it make sense to ask which integer is the smallest or which integer is the largest? Use a complete sentence to explain why or why not.

Investigate Problem 2

1. Math Path: Properties of Numbers and Their Operations

When we perform operations such as addition or multiplication on the numbers in a set, the operations could produce a number that is also in the system. When this happens, the set is said to be *closed* under the operation.

The set of integers is said to be closed under the operation of addition. This means that for every two integers a and b, the sum $a + b$ is also an integer.

Are the natural numbers closed under addition? Write an example to support your answer.

Are the whole numbers closed under addition? Write an example to support your answer.

Investigate Problem 2

2. Consider the operation of subtraction. Are the natural numbers closed under subtraction? Write an example to support your answer.

Are the whole numbers closed under subtraction? Write an example to support your answer.

Are the integers closed under subtraction? Write an example to support your answer.

Are any of these sets closed under multiplication? Write examples to support your answers.

Are any of these sets closed under division? Write examples to support your answer.

3. In earlier lessons, we learned about the additive inverse, the multiplicative inverse, the additive identity, and the multiplicative identity.

Which of these does the set of natural numbers have, if any? Use a complete sentence to explain.

Which of these does the set of whole numbers have, if any? Use a complete sentence to explain.

Which of these does the set of integers numbers have, if any? Use a complete sentence to explain.

Problem 3 Rational Numbers

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where *a* and *b* are both integers and *b* is not equal to 0.

A. Does the set of rational numbers include the set of whole numbers? Write an example to support your answer.

$\square = \frac{\square}{\square}$

B. Does the set of rational numbers include the set of integers? Write an example to support your answer.

$\square = \frac{\square}{\square}$

C. Does the set of rational numbers include all fractions? Write an example to support your answer.

$\frac{\square}{\square}$

D. Does the set of rational numbers include all decimals? Write an example to support your answer.

$\square = \frac{\square}{\square}$

We will examine the answer to Part (D) in more detail in Lesson 14.3. In that lesson we will determine which decimal numbers can be written exactly as fractions.

Investigate Problem 3

1. Is the set of rational numbers closed under addition? Write an example to support your answer.

Is the set of rational numbers closed under subtraction? Write an example to support your answer.

Is the set of rational numbers closed under multiplication? Write an example to support your answer.

Is the set of rational numbers closed under division? Write an example to support your answer.

Investigate Problem 3

2. Does the set of rational numbers have an additive identity? Write an example to support your answer.

Does the set of rational numbers have a multiplicative identity? Write an example to support your answer.

Does the set of rational numbers have an additive inverse? Write an example to support your answer.

Does the set of rational numbers have a multiplicative inverse? Write an example to support your answer.

3. You can add, subtract, multiply, and divide rational numbers in much the same way that you did using integers. Perform the indicated operation.

$1.5 + (-8.3) =$

$-12.5 - 8.3 =$

$-\frac{1}{2} - \frac{3}{4} =$

$2\frac{1}{2} + \left(-3\frac{7}{8}\right) =$

$-2.0 \times (-3.6) =$

$6.75 \times (-4.2) =$

$-\frac{2}{3} \times \frac{3}{8} =$

$-3\frac{3}{4} \times \left(-2\frac{3}{5}\right) =$

$-1.5 \div 4.5 =$

$-2.1 \div (-3.5) =$

$-\frac{2}{5} \div \frac{3}{10} =$

$-1\frac{3}{8} \div \left(-2\frac{2}{5}\right) =$

14.2 How Many Times?

Powers of Rational Numbers

Objectives

In this lesson, you will:

- Find powers of rational numbers.
- Multiply and divide powers of rational numbers.

Key Terms

- power

In Lesson 1.6, we learned that a **power** is a number written using a base and an exponent that represents repeated multiplication.

$$\underbrace{\overset{\text{base}}{4}^{\overset{\text{exponent}}{5}}}_{\text{power}} = 4 \times 4 \times 4 \times 4 \times 4$$

Problem 1 How Many Times More Cellular Phones?

In 1985, there were about 5^8 people in the United States who owned a cellular phone. By 2002, that number had grown to about 5^{12} people. How many times greater was the number of people who owned a cellular phone in 2002 than in 1985?

A. To find the value of each **power**, you can write 5^8 and 5^{12} as repeated multiplication and multiply.

$5^8 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 =$

$5^{12} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 =$

To answer the question, you could divide the value of 5^{12} by the value of 5^8 above, but in this lesson, we will learn an easier way!

B. The number of people who owned cellular phones in 1985 is actually closer to $\left(\frac{49}{10}\right)^8$ people. To find this number, let's begin by finding the values of some easier powers by writing each power first as repeated multiplication and then multiplying.

$\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right) =$

$\left(\frac{2}{3}\right)^3 =$

$\left(\frac{2}{3}\right)^4 =$

$\left(\frac{2}{3}\right)^5 =$

C. What can you conclude about finding the value of a power whose base is a rational number? Write a complete sentence to explain.

Investigate Problem 1

1. Use the results of Part (C) of to complete each statement.

$$\left(\frac{3}{4}\right)^3 = \frac{3^{\square}}{4^{\square}} = \frac{\square}{\square} \qquad \left(\frac{1}{6}\right)^3 = \frac{1^{\square}}{6^{\square}} = \frac{\square}{\square} \qquad \left(\frac{3}{5}\right)^4 = \frac{3^{\square}}{5^{\square}} = \frac{\square}{\square}$$

2. Find the value of each power by first writing it as repeated multiplication and then multiplying.

$(-1)^3 =$ $\qquad$ $(-1)^4 =$ $\qquad$ $(-1)^5 =$

$(-1)^6 =$ $\qquad$ $(1)^3 =$ $\qquad$ $(1)^{100} =$

3. What can you conclude about finding the value of a power of –1? What can you conclude about finding the value of a power of 1? Write complete sentences to explain.

4. Use your results from Part (B) and Question 2 to find the value of each power.

$\left(-\frac{2}{3}\right)^2 =$ $\qquad$ $\left(-\frac{2}{3}\right)^3 =$

$\left(-\frac{2}{3}\right)^4 =$ $\qquad$ $\left(-\frac{2}{3}\right)^5 =$

5. In Lesson 7.7, we learned about negative exponents. Recall that

$$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}.$$

Use what you have learned to complete each statement to find the value of the power.

$$(2)^{-3} = \frac{1}{2^{\square}} = \frac{\square}{\square} \qquad (10)^{-3} = \frac{1}{10^{\square}} = \frac{\square}{\square} \qquad (-3)^{-2} = \frac{1}{(-3)^{\square}} = \frac{\square}{\square}$$

6. Use what you have learned from Questions 1 and 5 to complete each statement.

$$\left(\frac{5}{6}\right)^{-2} = \frac{5^{\square}}{6^{\square}} = \frac{\frac{1}{5^{\square}}}{\frac{1}{6^{\square}}} = \frac{\frac{1}{\square}}{\frac{1}{\square}} = \frac{\square}{\square}$$

$$\left(\frac{3}{4}\right)^{-3} = \frac{3^{\square}}{4^{\square}} = \frac{\frac{1}{3^{\square}}}{\frac{1}{4^{\square}}} = \frac{\frac{1}{\square}}{\frac{1}{\square}} = \frac{\square}{\square}$$

Problem 2 Multiplying Powers

A. Find the value of each product of powers by first writing each power as repeated multiplication and then multiplying.

$(3)^2(3)^3 =$

$(2)^3(2)^4 =$

$(-5)(-5)^3 =$

$(-10)^4(-10)^3 =$

$\left(\frac{1}{4}\right)^2\left(\frac{1}{4}\right)^3 =$

$\left(-\frac{4}{5}\right)^2\left(-\frac{4}{5}\right)^3 =$

B. What conclusion can you make about multiplying powers? Write your answer using a complete sentence.

C. Complete the statement below to illustrate the conclusion that you wrote in Part (B).

$a^b a^c = a^{\square}$

D. Do you think that this rule applies to multiplying powers with integer exponents? To decide, first find the value of the product of powers below by writing each power as repeated multiplication and multiplying.

$$(5)^{-2}(5)^{-1} = \left(\frac{1}{5^{\square}}\right)\left(\frac{1}{5^{\square}}\right) = \left(\frac{1}{\square}\right)\left(\frac{1}{\square}\right) = \frac{1}{\square}$$

Apply your rule from part (C) to find the value of the product.

$$(5)^{-2}(5)^{-1} = 5^{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

E. Use the example above to write a complete sentence that explains your answer.

Investigate Problem 2

1. Find the value of each product of powers.

$(3)^{-2}(3)^3 =$

$(2)^{-2}(2)^{-3} =$

$(-1)^{-5}(-1)^3 =$

$(10)^{-5}(-10)^5 =$

$\left(\frac{3}{4}\right)^{-2}\left(\frac{3}{4}\right)^{-2} =$

$\left(-\frac{1}{2}\right)^{-5}\left(-\frac{1}{2}\right)^7 =$

Problem 3 Dividing Powers

A. Find the value of each quotient of powers by first writing the division problem as a multiplication problem using integer exponents. Then apply what you learned in Problem 2 to multiply the powers. The first one is done for you.

$$\frac{(3)^2}{(3)^3} = (3)^2 \times (3)^{-3} = 3^{2+(-3)} = 3^{-1} = \frac{1}{3}$$

$$\frac{(-5)^3}{(-5)} =$$

B. What conclusion can you make about dividing powers? Write your answer using a complete sentence.

C. Complete the equation below to illustrate the conclusion that you wrote in Part (B).

$$\frac{a^b}{a^c} = a^{\boxed{}}, \; a \neq 0$$

Investigate Problem 3

1. Find the value of each quotient of powers.

$$\frac{(-3)^3}{(-3)^2} = \qquad \frac{(3)^{-2}}{(3)^3} =$$

$$\frac{(-10)^4}{(-10)^{-3}} = \qquad \frac{(2)^{-2}}{(2)^{-3}} =$$

$$\frac{(10)^{-5}}{(10)^5} = \qquad \frac{\left(-\frac{4}{5}\right)^3}{\left(-\frac{4}{5}\right)^2} =$$

2. Remember in Problem 1 that we wanted to find out how many times greater the number of cellular phone owners was in 2002 than in 1985. We can find this number by writing the quotient $\frac{5^{12}}{5^8}$. Use what you learned in Problem 3 to find this quotient.

Do you agree that this is much easier than multiplying out the numerator and denominator and then dividing?

14.3 Sew What?

Irrational Numbers

Objectives

In this lesson, you will:

- Identify decimals as terminating or repeating.
- Write repeating decimals as fractions.
- Identify irrational numbers.

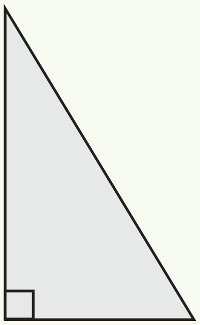
Key Terms

- terminating decimal
- repeating decimal
- bar notation
- irrational number

You have worked with some numbers that are not rational numbers. For example, $\sqrt{2}$ and $\sqrt{5}$ are not the square roots of perfect squares and cannot be written in the form $\frac{a}{b}$, where a and b are both integers.

Problem 1 Making a Quilt

Your aunt is making a quilt. The pattern pieces are right triangles with the dimensions shown.



A. Is it possible to exactly measure the length of the hypotenuse? Use the Pythagorean theorem by filling in the blanks below. Then use a complete sentence to explain your answer.

$$a^2 + b^2 = c^2$$

$$2^2 + 3^2 = c^2$$

$$\boxed{} = c^2$$

$$\sqrt{\boxed{}} = c$$

B. Even though we often approximate square roots using a decimal, a decimal representing a square root that is not a perfect square is an *irrational number.* Because all fractions are rational numbers, decimals that can be written as fractions are rational numbers. Other decimals are irrational numbers.

Convert the fraction to a decimal by dividing the numerator by the denominator. Continue to divide until you see a pattern.

$\frac{1}{3} =$ $\qquad 3\overline{)1}$

C. Use a complete sentence to describe the pattern that you observed in Part (B).

Problem 1 Making a Quilt

D. Convert each fraction to a decimal by dividing the numerator by the denominator. Continue to divide until you see a pattern.

$\frac{5}{6} =$ $6\overline{)5}$

$\frac{2}{9} =$ $9\overline{)2}$

$\frac{9}{11} =$ $11\overline{)9}$

$\frac{3}{22} =$ $22\overline{)3}$

E. These decimal representations are called *repeating decimals.* Use a complete sentence to explain why.

Investigate Problem 1

1. Math Path: Terminating and Repeating Decimals

A **terminating decimal** is a decimal that has a last digit. For instance, the decimal $0.125 = \frac{125}{1000} = \frac{1}{8}$ is a terminating decimal.

A **repeating decimal** is a decimal with digits that repeat in sets of one or more. We can use two different notations to represent repeating decimals. One notation is to write the decimal, including one set of digits that repeat, and place a bar over the repeating digits. This is called **bar notation.**

$\frac{1}{3} = 0.\overline{3}$

$\frac{7}{22} = 0.3\overline{18}$

Another notation is to write the decimal, including two sets of the digits that repeat, and using dots to indicate repetition.

$\frac{1}{3} = 0.33\ldots$

$\frac{7}{22} = 0.31818\ldots$

Write each repeating decimal that you found in Part (D) using both notations.

$\frac{5}{6} =$

$\frac{2}{9} =$

$\frac{9}{11} =$

$\frac{3}{22} =$

Investigate Problem 1

2. Some repeating decimals represent common fractions, such as $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{6}$, and are used often enough that we recognize the fraction by its decimal representation. For most repeating decimals, though, you cannot recognize the fraction that the decimal represents. For instance, can you tell which fraction is represented by the repeating decimals 0.44… or $0.\overline{09}$?

Both of these repeating decimals represent fractions that you have used. We can use algebra to find the fraction that is represented by the repeating decimal 0.44… . First, write an equation by setting the decimal equal to a variable that will represent the fraction.

$w = 0.44...$

Next, multiply both sides of the equation by a power of 10. The exponent on the power of 10 is equal to the number of decimal places until the decimal begins to repeat. In this case, the decimal begins repeating after 1 decimal place, so the exponent on the power of 10 is 1. Because $10^1 = 10$, multiply both sides by 10.

$10w = 4.4...$

Next, subtract the first equation from the second equation.

$$\begin{array}{r} 10w = 4.44... \\ -w = 0.44... \\ \hline 9w = 4 \end{array}$$

Finally, solve the equation by dividing both sides by what number? So, what fraction is w?

3. Complete the steps to find the fraction that is represented by 0.0909... .

$w = 0.0909...$

$100w = 9.0909...$

$$\begin{array}{r} 100w = 9.0909... \\ -w = 0.0909... \\ \hline \square\ w = \square \\ w = \frac{\square}{\square} = \frac{\square}{\square} \end{array}$$

4. Repeat the procedure above to write the fraction that represents each repeating decimal.

0.55... =

0.0505... =

$0.\overline{12} =$

$0.\overline{36} =$

Problem 2 Sewing a Tablecloth

Your aunt wants to sew a round tablecloth with lace trim. The diameter of the tablecloth must be 70 inches. Your aunt wants to know how much trim to purchase.

A. In Lesson 10.2, we found the circumference of a circle. What number should your aunt multiply the diameter of the tablecloth by in order to know how many inches of lace trim to purchase? Write your answer using a complete sentence.

B. In Lesson 10.2, we used an approximation of the number π to find the circumference of a circle. Even though we used this approximation, the number π is a decimal with a never-ending number of digits that do not repeat. Decimals that do not repeat and do not terminate are said to be *irrational numbers.* An **irrational number** is a number that cannot be written in the form $\frac{a}{b}$, where a and b are both integers and $b \neq 0$.

C. The most famous irrational number is π. Throughout history, π has been calculated to hundreds and thousands of decimal places. In fact, one way that the speed of computers is measured is based on how fast the computer can calculate the digits of π. Another irrational number is $\sqrt{6}$. This number must be between which two whole numbers?

Use a calculator find each power.

$2.1^2 =$ $2.2^2 =$ $2.3^2 =$

$2.4^2 =$ $2.5^2 =$ $2.6^2 =$

Which of the bases above must $\sqrt{6}$ be between?

Continue the process above for the hundredths place.

$2.41^2 =$ $2.42^2 =$ $2.43^2 =$

$2.44^2 =$ $2.45^2 =$ $2.46^2 =$

Continue the process above for the thousandths place. Are the numbers you are squaring getting close to $\sqrt{6}$?

$2.445^2 =$ $2.446^2 =$ $2.447^2 =$

$2.448^2 =$ $2.449^2 =$ $2.450^2 =$

D. Find $\sqrt{6}$ using the calculator. Do you see any digits repeating in a pattern?

What can you conclude about the square roots of numbers that are not perfect squares? Write your answer using a complete sentence.

14.4 Worth 1000 Words

Real Numbers and Their Properties

Objectives

In this lesson, you will:

- Classify numbers in the real number system.
- Understand the properties of real numbers.

Key Terms

- real number
- Venn diagram

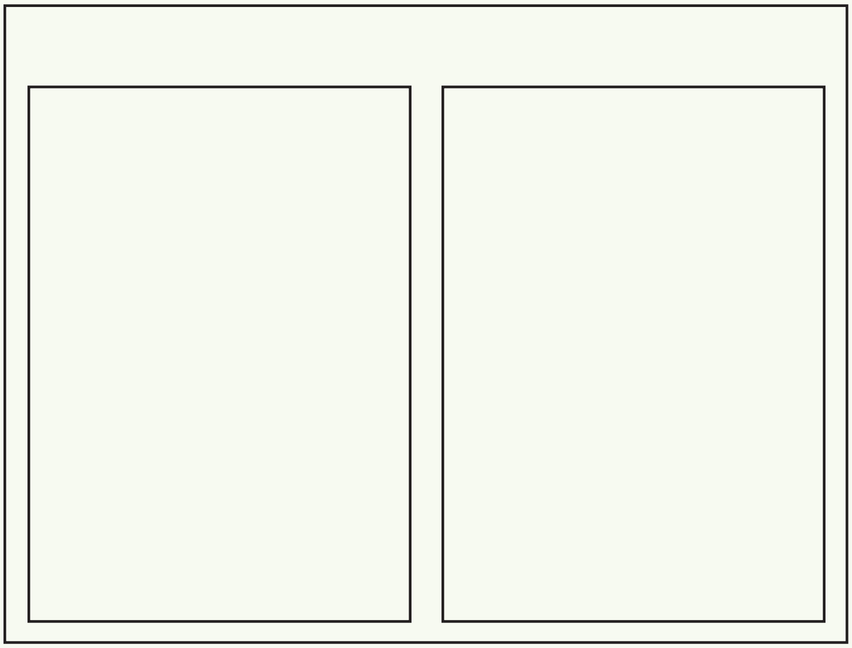
Take Note

The Venn diagram was introduced in 1881 by John Venn, British philosopher and mathematician.

Problem 1 Picturing the Real Numbers

Combining the set of rational numbers and the set of irrational numbers produces the set of **real numbers.** You can use a **Venn diagram** to represent how the sets within the set of real numbers are related.

A. First, at the top of the large rectangle, write the label "Real Numbers." This entire rectangle represents the set of real numbers.



B. Label the smaller rectangle at the right "Irrational Numbers."

C. Label the top of the smaller rectangle at the left "Rational Numbers."

D. Inside the rectangle that represents rational numbers, draw a large circle. Inside the circle, at its top, write the label "Integers."

E. Inside the circle that represents integers, draw a smaller circle. Inside the circle, at its top, write the label "Whole Numbers."

F. Inside the circle that represents the whole numbers, draw a smaller circle. Inside this circle, write the label "Natural Numbers." Your Venn diagram that represents the real number system is complete.

Investigate Problem 1

Use your Venn diagram in Problem 1 to decide whether each of the following statements is true or false. Write complete sentences to explain your reasoning.

1. A whole number is sometimes an irrational number.
2. A real number is sometimes a rational number.
3. A whole number is always an integer.
4. A negative integer is always a whole number.
5. A rational number is sometimes an integer.
6. A square root is always an irrational number.
7. A fraction is never an irrational number.
8. A decimal is sometimes an irrational number.

Problem 2 Properties of Real Numbers

The real numbers, together with their operations and properties, form the real number system. You have already encountered many of the properties of the real number system in various lessons. Let's review these properties.

A. **Closure:** A set of numbers is said to be closed under an operation if the result of the operation on two numbers in the set is another member of the set. For instance, the set of integers is closed under addition. This means that for every two integers a and b, the sum $a + b$ is also an integer.

For any real numbers a and b, is $a + b$ a real number? Is the set of real numbers closed under addition? Write an example to support your answer.

For any real numbers a and b, is $a - b$ a real number? Is the set of real numbers closed under subtraction? Write an example to support your answer.

For any real numbers a and b, is $a \times b$ a real number? Is the set of real numbers closed under multiplication? Write an example to support your answer.

For any real numbers a and b, is $a \div b$ a real number? Is the set of real numbers closed under division? Write an example to support your answer.

B. **Additive Identity:** An additive identity is a number such that when you add it to a second number, the sum is equal to the second number.

For any real number a, is there a real number such that a + (the number) = a? What is the number?

Does the set of real numbers have an additive identity? Write an example to support your answer.

C. **Multiplicative Identity:** A multiplicative identity is a number such that when you multiply it by a second number, the product is equal to the second number.

For any real number a, is there a real number such that a × (the number) = a? What is the number?

Does the set of real numbers have a multiplicative identity? Write an example to support your answer.

Problem 2 Properties of Real Numbers

D. Additive Inverse: Two numbers are additive inverses if their sum is the additive identity.

For any real number a, is there a real number such that $a + (\text{the number}) = 0$? What is the number?

Does the set of real numbers have an additive inverse? Write an example to support your answer.

E. Multiplicative Inverse: Two numbers are multiplicative inverses if their product is the multiplicative identity.

For any real number a, is there a real number such that $a \times (\text{the number}) = 1$? What is the number?

Does the set of real numbers have a multiplicative inverse? Write an example to support your answer.

F. Commutative Property of Addition: Changing the order of two or more addends in an addition problem does not change the sum.

For any real numbers a and b, is $a + b = b + a$? Write an example to support your answer.

G. Commutative Property of Multiplication: Changing the order of two or more factors in a multiplication problem does not change the product.

For any real numbers a and b, is $a \times b = b \times a$? Write an example to support your answer.

H. Associative Property of Addition: Changing the grouping of the addends in an addition problem does not change the sum.

For any real numbers a, b and c, is $(a + b) + c = a + (b + c)$? Write an example to support your answer.

I. Associative Property of Multiplication: Changing the grouping of the factors in a multiplication problem does not change the product.

For any real numbers a, b, and c, is $(a \times b) \times c = a \times (b \times c)$? Write an example to support your answer.

Problem 2 Properties of Real Numbers

J. Properties of Equality:

Reflexive Property of Equality:

For any real number a, $a = a$.

Write an example of the property.

Symmetric Property of Equality:

For any real numbers a and b, if $a = b$, then $b = a$.

Write an example of the property.

The Transitive Property of Equality:

For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Write an example of the property.

Choose one of the properties of equality and write a complete sentence to explain the property.

Investigate Problem 2

1. For each problem, identify the property that is represented.

$234 + (-234) = 0$

$-4 \times (3 \times 5) = (-4 \times 3) \times 5$

$-24 \times 1 = -24$

$-67 \times 56 = 56 \times (-67)$

$-456 + 34 = 34 + (-456)$

$4 \times 0.25 = 1$

If $5 = (-1)(-5)$ then $(-1)(-5) = 5$.

If $c = 5 \times 7$ and $35 = 70 \div 2$, then $c = 70 \div 2$.

$a + (4 + c) = (a + 4) + c$

$\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 1$

$-2\frac{3}{4} \times 1 = -2\frac{3}{4}$

$\left(-\frac{3}{4}\right) + \left(\frac{4}{3} + 5\right) = \left(-\frac{3}{4} + \frac{4}{3}\right) + 5$

14.5

The House That Math Built

The Distributive Property

Objectives

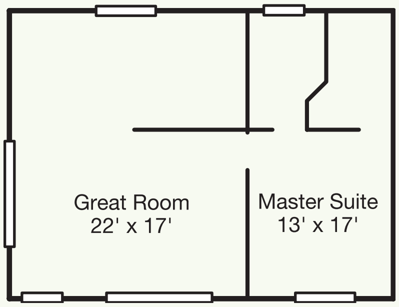
In this lesson, you will:

- Understand the distributive property.

Key Terms

- distributive property

Problem 1 Carpeting



As a contractor, you need to determine the number of square feet of carpeting that you need to cover the great room and the master suite shown in the floor plan. The great room is 22 feet by 17 feet and the master suite is 13 feet by 17 feet.

A. How would you help the contractor find the area of the great room and the master suite? Write your answer using a complete sentence.

B. In Part (A), there are two ways to find the area of the great room and the master suite. You could find the area of each room and then add to get the total area.

Total area in square feet $= (17 \times 22) + (17 \times 13)$

$= \boxed{} + \boxed{} = \boxed{}$

Or, you could multiply the width by the total length of both rooms.

Total area in square feet $= 17 \times (22 + 13)$

$= 17 \times \boxed{} = \boxed{}$

C. Finding the area in these two ways is an example of the **distributive property.** This property is useful in simplifying complicated expressions. The *Distributive Property of Multiplication Over Addition* distributes multiplication over addition. It says that you can multiply a number and a sum by first multiplying the number by each addend of the sum and then adding the products. For example, $5(3 + 7) = 5(3) + 5(7)$.

Find the value of the left side of the equation above by adding and then multiplying.

$5(3 + 7) = 5(\boxed{}) = \boxed{}$

Find the value of the right side of the equation above by multiplying and then adding.

$5(3) + 5(7) = \boxed{} + \boxed{} = \boxed{}$

Investigate Problem 1

1. The *Distributive Property of Multiplication Over Subtraction* distributes multiplication over subtraction. It says that you can multiply a number and a difference by first multiplying the number by each part of the difference and then subtracting the products. Write an equation that illustrates this property below. Then find the value of each side of the equation.

2. The *Distributive Property of Division Over Addition* distributes division over addition. It says that you can divide a sum by a number by first dividing each addend by the number and then adding the quotients. Write an equation that illustrates this property below. Then find the value of each side of the equation.

3. The *Distributive Property of Division Over Subtraction* distributes division over subtraction. It says that you can divide a difference by a number by first dividing each part of the difference by the number and then subtracting the quotients. Write an equation that illustrates this property below. Then find the value of each side of the equation.

Take Note

Remember that to evaluate an expression means to find the value of the expression.

4. Use the distributive property to evaluate each expression. Show your work.

$14(10 + 1) =$

$6(9 + x) =$

$-2(2 + 17) =$

$-7(5x + 6) =$

$4(20 - 6) =$

$9(x - 5) =$

$-2(14 - 8) =$

$-1(1 - x) =$

$\frac{(27 + 6)}{9} =$

$\frac{(125 + 25x)}{5} =$

$\frac{(300 - 21)}{3} =$

$\frac{(8x - 44)}{4} =$

$\frac{(-45 + 72)}{-9} =$

$\frac{(-36 - 6x)}{2} =$

Investigate Problem 1

5. For each equation, identify the property used in each step.
For example:

$5(3+7)-34=4(9-7)+8$	•	Given problem
$15+35-34=4(9-7)+8$	•	Distributive Property of Multiplication Over Addition
$15+35-34=36-28+8$	• •	Distributive Property of Multiplication Over Subtraction
$(15+35)-34=(36-28)+8$	•	Group
$50-34=8+8$	•	Addition and subtraction
$16=16$	•	Reflexive Property of Equality

$2(8x+89)=\frac{(24-8)}{8}$ ____________________

$16x+178=\frac{(24-8)}{8}$ ____________________

$16x+178=\frac{24}{8}-\frac{8}{8}$ ____________________

$16x+178=3-1$ ____________________

$16x+178=2$ ____________________

$16x+178-178=2-178$ ____________________

$16x=-176$ ____________________

$\frac{16x}{16}=\frac{-176}{16}$ ____________________

$x=-11$ ____________________

$20+(x+6)+7=3(6+5)+x$ ____________________

$20+x+(6+7)=3(6+5)+x$ ____________________

$20+x+13=3(6+5)+x$ ____________________

$x+(20+13)=3(6+5)+x$ ____________________

$x+33=3(6+5)+x$ ____________________

$x+33=3(6)+3(5)+x$ ____________________

$x+33=18+15+x$ ____________________

$x+33=(18+15)+x$ ____________________

$x+33=33+x$ ____________________

$x+33=x+33$ ____________________

Looking Back at Chapter 14

Key Terms

natural number ● p. 462
whole number ● p. 462
integer ● p. 463
rational number ● p. 465
power ● p. 467
terminating decimal ● p. 472
repeating decimal ● p. 472
bar notation ● p. 472
irrational number ● p. 474
real number ● p. 475
Venn diagram ● p. 475
distributive property ● p. 481

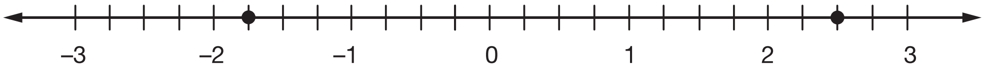
Summary

Comparing Rational Numbers Using a Number Line (p. 461)

To compare rational numbers, write each rational number as a decimal and plot the numbers on a number line. The number furthest to the right is greatest number.

Example To compare the numbers $\frac{5}{2}$ and $-\frac{7}{4}$, rewrite each rational number as a decimal. Then plot each decimal on a number line.

$\frac{5}{2} = 2.5$ $-\frac{7}{4} = -1.75$



So, $\frac{5}{2}$ is greater than $-\frac{7}{4}$.

Performing Operations with Rational Numbers (p. 466)

To add, subtract, multiply, and divide rational numbers, use what you know about adding, subtracting, multiplying, and dividing integers and fractions.

Examples

$$3\frac{1}{7} + \left(-\frac{5}{7}\right) = \frac{22}{7} + \left(-\frac{5}{7}\right) = \frac{17}{7}$$

$$-\frac{4}{3} \div \frac{7}{5} = -\frac{4}{3} \times \frac{5}{7} = -\frac{20}{21}$$

Finding Powers of Rational Numbers (p. 467)

To find the value of a power whose base is a rational number, use the rule $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$.

Examples

$$\left(\frac{4}{7}\right)^2 = \frac{4^2}{7^2} = \frac{16}{49}$$

$$\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5} = \frac{243}{32}$$

$$\left(\frac{9}{14}\right)^{-3} = \frac{9^{-3}}{14^{-3}} = \frac{\frac{1}{9^3}}{\frac{1}{14^3}} = \frac{\frac{1}{729}}{\frac{1}{2744}} = \frac{2744}{729}$$

Finding Products of Powers (p. 469)

To find the product of powers, use the rule $a^b a^c = a^{b+c}$.

Examples $(4)^3(4)^5 = (4)^{3+5} = 4^8$ $\left(\frac{1}{8}\right)^2\left(\frac{1}{8}\right)^3 = \left(\frac{1}{8}\right)^5$ $(4)^{3+5} = 4^8$

$\left(\frac{3}{4}\right)^{-5}\left(\frac{3}{4}\right)^{-1} = \left(\frac{3}{4}\right)^{-5+(-1)} = \left(\frac{3}{4}\right)^{-6}$ $(12)^2(12)^{-9} = (12)^{-7}$

Finding Quotients of Powers (p. 470)

To find the quotient of powers, use the rule $\frac{a^b}{a^c} = a^{b-c}, a \neq 0$.

Examples $\frac{(5)^6}{(5)^3} = (5)^{6-3} = 5^3$ $\frac{(-4)^3}{(-4)^5} = (-4)^{3-5} = (-4)^{-2}$

$\frac{\left(\frac{2}{9}\right)}{\left(\frac{2}{9}\right)^{-2}} = \left(\frac{2}{9}\right)^{1-(-2)} = \left(\frac{2}{9}\right)^3$ $\frac{(7)^{-1}}{(7)^6} = (7)^{-1-6} = (7)^{-7}$

Identifying Decimals as Terminating or Repeating (p. 472)

To identify a decimal as terminating or repeating, determine whether the decimal has a last digit or digits that repeat in sets.

Examples $\frac{5}{8} = 0.625$ The decimal has a last digit 5, so it is a terminating decimal.

$\frac{2}{11} = 0.1818...$ The decimal repeats the set of digits “18,” so it is a repeating decimal. You can write this decimal using bar notation as $0.\overline{18}$.

Writing Repeating Decimals as Fractions (p. 473)

To write a repeating decimal as a fraction, first write an equation by setting the decimal equal to a variable that will represent the fraction. Then multiply both sides of the equation by a power of ten. Subtract the first equation from the second equation and solve.

Example To write 0.22… as a fraction, first write the equation $x = 0.22...$. Then multiply both sides by 10 and subtract the original equation. Finally, solve for x.

$$\begin{aligned} x &= 0.22... \\ 10x &= 2.22... \\ -x &= 0.22... \\ 9x &= 2 \\ x &= \frac{2}{9} \end{aligned}$$

So, the decimal 0.22… is equal to the fraction $\frac{2}{9}$.

Identifying Irrational Numbers (p. 474)

An irrational number is a number than cannot be written in the form $\frac{a}{b}$. Decimals that do not repeat and do not terminate are irrational numbers.

Examples

The number $\sqrt{2} = 1.4142135623...$ is an irrational number because it does not terminate and does not repeat.

The number 2.457457457457 is a rational number because it repeats.

Classifying Numbers (p. 475)

To classify a number in the real number system, determine whether the number belongs to the set of rational or irrational numbers. If the number is a rational number, then decide whether it belongs to the set of natural numbers, whole numbers, or integer numbers.

Examples

The number –3.7 is a real number belonging to the set of rational numbers.

The number 7 is a real number belonging to the set of rational numbers. This number also belongs to the set of integer numbers, the set of whole numbers, and the set of natural numbers.

The number 6.54384972469… is a real number belonging to the set of irrational numbers.

Identifying Properties of Real Numbers (p. 477)

Use the properties you have learned to identify the property that is being used in each statement.

Examples

For real numbers a and b, the number $a - b$ is a real number.

Closure: This statement is true because the set of real numbers is closed under subtraction.

$4 + (-4) = 0$

Additive Inverse: Two numbers are additive inverses if the sum of the numbers is the additive identity 0.

$(3 \times 5) \times 9 = 3 \times (5 \times 9)$

Associative Property of Multiplication: Changing the grouping of the factors in a multiplication problem does not change the product.

Using the Distributive Property (p. 481)

You can use the distributive property as another way to solve multiplication and division problems involving addition and subtraction.

Examples

$$\begin{aligned} 5(3+4) &= 5(3)+5(4) \\ &= 15+20 \\ &= 35 \end{aligned}$$

$$\begin{aligned} \frac{(34+6y)}{4} &= \frac{34}{4}+\frac{6y}{4} \\ &= \frac{17}{2}+\frac{3}{2}y \end{aligned}$$