Looking Ahead to Chapter 13

13

Focus In Chapter 13, you will work with linear functions and learn about function notation. You will use the slope and *x*- and *y*-intercepts to graph linear functions and to write linear equations in slope-intercept form. You will also create scatter plots and find lines of best fit.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 13.

Write each ratio as a fraction.

1. Two of my five pens are red.
2. Seventeen of the thirty math students are girls.

Write each rate as a fraction.

3. Paul used 20 gallons of gas to drive 500 miles.
4. A 13-ounce can of coffee costs $3.25.

Evaluate the expression when $x = 4$.

5. $4x + 2$
6. $5(x - 9)$
7. $\frac{x}{2} - 7$

Read the problem scenario below.

Kelly has $600 in the bank. She just bought a cellular phone and must pay $30 each month to her cellular phone company.

8. Using the variable *m* to represent months, write an expression to show how much money Kelly will have in the bank after she starts paying her cellular phone bill.
9. How much money will she have after 3 months?
10. How much money will Kelly have after 8 months?
11. How many months can she pay for her phone before she runs out of money?

Key Terms

relation • p. 419
input • p. 419
output • p. 419
function • p. 419
input-output table • p. 420
independent variable • p. 421
dependent variable • p. 421
domain • p. 422
range • p. 422
function notation • p. 422
linear function • p. 423
rise • p. 427
run • p. 427
slope • p. 427
rate of change • p. 430
x-intercept • p. 438
y-intercept • p. 438
linear equation • p. 444
slope-intercept form • p. 445
scatter plot • p. 450
line of best fit • p. 450

13

13

Linear Functions

Rock climbers who use only their hands, feet, and other body parts to make upward progress are "free-climbing," using ropes and other gear only for protection. In Lesson 13.2, you will use a linear function to determine your height at different times when rock climbing.

13

13.1 Running a Tree Farm

Relations and Functions

13

Objectives

In this lesson, you will:

- Use tables and graphs to represent functions.
- Use function notation.

Key Terms

- relation
- input
- output
- function
- input-output table
- independent variable
- dependent variable
- domain
- range
- function notation

In Chapter 8, when we were solving problems using multiple representations, we were actually establishing relationships between two quantities. In this lesson, we will also express relationships in many ways—with words, using an algebraic expression, making a table, and using a graph.

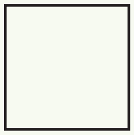
Problem 1 Planting Seedlings

A **relation** is the mathematical term for any set of ordered pairs. The first coordinate of an ordered pair in a relation is called the **input** and the second coordinate is called the **output.** A relation is a **function** if for every input you have exactly one output.

A. On your tree farm, you can find many situations in which a function would be helpful. For example, you might want to describe the relationship between the ages of the trees you have growing. We can express a function in several ways.

In words: The saplings in a new planting are four years younger than the trees in the previous planting.

Write the words to express the perimeter of the square planting box in terms of the side length of the box.



B. **Using an algebraic expression:** You can express the age of the saplings in a new planting as $t - 4$, where t is the age in years of the trees in the previous planting.

Suppose you measure two trees. The first tree's height is 3 feet more than 4 times the second tree's height. Write an expression to represent the first tree's height in terms of the second tree's height.

Write four ordered pairs that satisfy this relationship.

13

Problem 1 Planting Seedlings

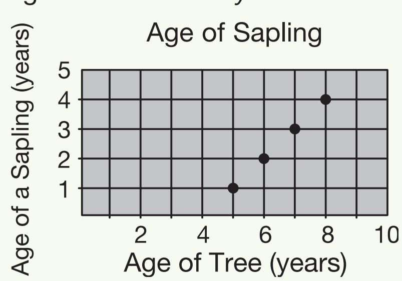
C. Making a table: The **input-output table** shows the relationship between the input value, the age of a tree in the previous planting, and the output value, the age of a sapling in a new planting.

Age of Tree (years)	Function $s = t - 4$	Age of Sapling (years)
5	$s = 5 - 4 = 1$	1
6	$s = 6 - 4 = 2$	2
7	$s = 7 - 4 = 3$	3
8	$s = 8 - 4 = 4$	4

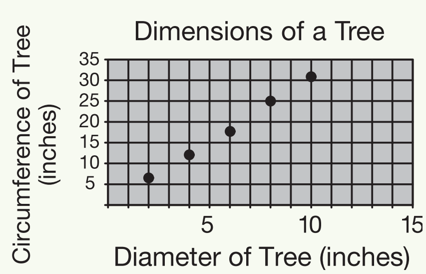
A tree farm is having a sale. The regular prices and the sale prices are shown in the table. Write each row of numbers in the table as an ordered pair. Then use a complete sentence to explain how the regular price and the sale price are related.

Regular Price	Sale Price
$20	$10
$30	$15
$50	$25
$200	$100

D. Using a graph: The graph shows the relationship between the age of a tree and the age of a sapling. The *x*-coordinate is the age of a tree. The *y*-coordinate is the age of a sapling.



E. The values in the table at the left are plotted as ordered pairs in the graph at the right. Use a complete sentence to explain how the circumference of a tree is related to its diameter.



x	*y*
2	6.28
4	12.57
6	18.85
8	25.13
10	31.40

Investigate Problem 1

1. A function is a special relation in which every x-coordinate is paired with one and only one y-coordinate. A function relates two quantities, so it is often written as a mathematical sentence using two variables. For instance, you can write "The first tree's height is 3 more than 4 times the second tree's height" as $y = 3 + 4x$, where x represents the second tree's height and y represents the first tree's height.

   Decide whether each relation below is a function. In each case, write a complete sentence explaining your reasoning.

   The cost of any item is 1.07 times its sales price.

   One number is the square root of another number.

   (3, 4), (6, 7), (–2, –7), (–3, –7)

   (–3, 5), (9, 2), (–5, –7), (–3, –9)

2. The word "function" is often used to describe a relationship. For example, a tree's height is a function of its age. The number of trucks we will need is a function of the number of trees we will be hauling. With most functions, the two quantities are related in such a way that the value of one quantity depends on the value of the other quantity. The **independent variable** represents the input values of the function and the **dependent variable** represents the output values. For each function below, identify the independent variable and the dependent variable.

   The amount of money that a tree farmer earns is a function of the number of trees that he sells.

   The water height in an irrigation pond rises 2 inches for every day with a good rain.

   A tree grows 3 feet each month.

   Your salary at a tree farm is $30,000 per year and increases by $600 each year.

   Seedlings cost $29.95 per case of one hundred with a shipping charge of $10.

Investigate Problem 1

3. Math Path: Domain and Range

For a function, the set of all input values is given the mathematical name **domain** and the set of all output values is given the name **range.** Suppose that you are buying fertilizer for the tree farm. Each bag of fertilizer covers 1000 square feet. You can haul up to 6 bags in your truck. The input-output table for the function that relates the number of bags of fertilizer to the number of square feet covered is shown below.

Bags of Fertilizer	Coverage (square feet)
1	1000
2	2000
3	3000
4	4000
5	5000
6	6000

The function that describes this relationship is $y = 1000x$, where x is the number of bags of fertilizer and y is the number of square feet covered.

What is the domain of the function?

What is the range of the function?

Take Note

In the function notation at the right, f is the name of the function, x represents the domain, and $f(x)$ represents the range.

4. Math Path: Function Notation

In addition to writing functions in terms of the independent and dependent variables, we can also use **function notation** to write a function. For instance, we can write the function $y = 1000x$ using function notation as $f(x) = 1000x$.

You read the symbol $f(x)$ as "f of x" or "the function of f at x." Rewrite each function below using function notation.

$y = x + 12$ $\qquad$ $y = 0.75x$ $\qquad$ $y = 4$

You can find the value of a function for a certain number by substituting the number into the function. To find the value of $f(x) = 1000x$ when $x = 6$, substitute 6 for x and simplify:

$f(x) = 1000x$
$f(6) = 1000(6)$
$f(6) = 6000$

Find the value of each function when $x = 5$.

$f(x) = 50x$	$f(x) = 9 - x$	$f(x) = x - 10$
$f(5) =$	$f(5) =$	$f(5) =$

13.2 Scaling a Cliff

Linear Functions

Objectives

In this lesson, you will:

- Make input-output tables for linear functions.
- Graph linear functions.

Key Terms

- linear function

When you graph the input and output values of some functions, the graph forms a straight line. A function whose graph is a straight line is a **linear function.** In this lesson, we will examine linear functions to determine some of their properties.

Problem 1 Rock Climbing

You and your friends are rock climbing a vertical cliff along a beach. You have been climbing for a while and are currently 36 feet above the beach when you stop on a ledge to have a snack and then begin climbing again. You can climb about 12 feet in height each hour.

A. How high will you have climbed above the beach in 1 hour after you begin climbing again? Use a complete sentence in your answer.

B. How high will you have climbed above the beach in 2 hours after you begin climbing again?

C. How high will you have climbed above the beach in 180 minutes after you begin climbing again? Use a complete sentence in your answer.

D. How high will you have climbed above the beach in 210 minutes after you begin climbing again?

E. Which quantities are changing? Which quantities remain constant? Use complete sentences in your answers.

F. Which quantity depends on the other quantity? Use a complete sentence in your answer.

Investigate Problem 1

1. Complete the table below by first filling in the names of the quantities in Problem 1 and their units of measure. Then fill in the table with your answers from Problem 1. The rest of the table will be completed later.

	Column 1	Column 2
Quantity Name		
Unit of Measure		
Problem 1, Part (A)	1	
Problem 1, Part (B)	2	
Problem 1, Part (C)		
Problem 1, Part (D)		
Question 4		84
Question 5		96
Question 6		108
Expression		

2. Define a variable for the quantity in Column 1. Enter this variable in the "Expression" row under Column 1.

3. Write an expression that you can use to represent the quantity in Column 2 in terms of the quantity in Column 1. Enter this expression in the "Expression" row under Column 2.

4. How long will it be until you have climbed to 84 feet above the beach? Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

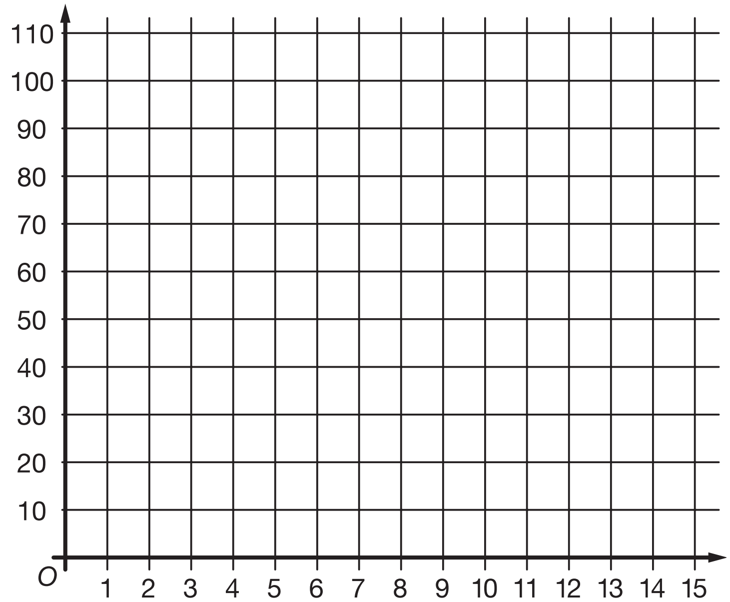
5. How long will it be until you have climbed to 96 feet above the beach? Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

6. How long will it be until you have climbed to 108 feet above the beach? Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

Investigate Problem 1

13

7. You will use the grid below to create a graph to represent the values in the table. Begin by labeling your axes. Use the first quantity from the table for the horizontal axis and the second quantity for the vertical axis. The axes are already numbered.



8. Write each row of numbers in your table as an ordered pair. Note that the *x*-coordinate is the amount of time that you have been climbing since you started climbing after your snack and the *y*-coordinate is the total height you have climbed so far.

9. Plot each ordered pair on the prepared grid in Question 7.

10. What do you notice about the points you graphed on the coordinate plane above? Write your answer using a complete sentence.

11. Draw a straight line through the points. How would you describe this line? Write your answer using a complete sentence.

Investigate Problem 1

12. Is the relation shown in the graph a function? Use a complete sentence to explain why or why not.

13. Is the relation shown in the graph a linear function? Use a complete sentence to explain why or why not.

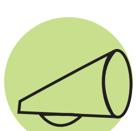
14. Which variable is the dependent variable? Write your answer using a complete sentence.

15. Which variable is the independent variable? Write your answer using a complete sentence.

16. Describe what happens to the value of the dependent variable each time the independent variable increases by 1. Use a complete sentence in your answer.

17. Describe what happens to the value of the dependent variable when the independent variable increases by 2. Use a complete sentence in your answer.

18. Compare the values of the dependent variable when the independent variable is 1 and 6. Describe how the dependent variable changes in relation to the independent variable.



19. Form a group with another partner team and compare your graphs and your answers to Questions 12 through 18. If you have any answers on which you do not agree, work together to find out why. Be prepared to share your answers with the rest of the class.

13.3 Biking Along

Slope and Rates of Change

13

Objectives

In this lesson, you will:

- Find the slope of a line as a ratio.
- Find the slope of a line as a rate of change.

Key Terms

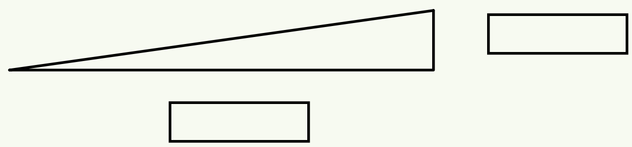
- rise
- run
- slope
- rate of change

In Lesson 13.2, we worked with linear functions. In this lesson, we will explore some of the special properties of linear functions.

Problem 1 Biking Uphill

You and your friend are taking a week-long trip on your bicycles. On the first day, you come to a hill that has a sign that says "8% grade." You tell your friend that this means that the hill has a vertical change of 8 feet for every 100 feet change in horizontal distance. The vertical change is the **rise** of the hill and the horizontal change is the **run.**

A. The diagram shows the grade of the hill. Fill in the boxes to indicate the rise and the run of the hill.



B. As you go up the hill, the ratio of the vertical change to the horizontal change has a special name. It is called the **slope** of the hill. Complete the statement to find the slope of the line that represents the hill.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\square \text{ feet}}{\square \text{ feet}} = \frac{\square}{\square}$$

C. When you are biking up this hill, the slope of the line that represents the hill is positive. After a short break at the top of the hill, you bike down the other side of the hill. Describe the sign of the slope of the line that represents your ride downhill. Use complete sentence to explain your reasoning.

D. Use the definition of slope to find the slope of the line that represents when you are biking on ground that is completely level. Use a complete sentence to explain.

13

Problem 2 Setting the Pace

You and your friend know from previous bicycle trips that you can bike at a rate of about 8 miles per hour.

A. How far will you go in 2 hours? Use a complete sentence in your answer.

B. How far will you go in 3 hours? Use a complete sentence in your answer.

C. How far will you go in 6 hours? Use a complete sentence in your answer.

D. How far will you go in 420 minutes? Use a complete sentence in your answer.

E. Which quantities are changing? Which quantities remain constant? Use complete sentences in your answers.

F. Which quantity depends on the other quantity? Use a complete sentence in your answer.

G. Complete the table below by first filling in the names of the quantities and their units of measure. Then fill in the table with your answers from Parts (A) through (D). The rest of the table will be completed later.

	Column 1	Column 2
Quantity Name		
Unit of Measure		
Problem 2, Part (A)	2	
Problem 2, Part (B)	3	
Problem 2, Part (C)	6	
Problem 2, Part (D)		
Question 3		44
Question 4		164
Expression		

Investigate Problem 2

13

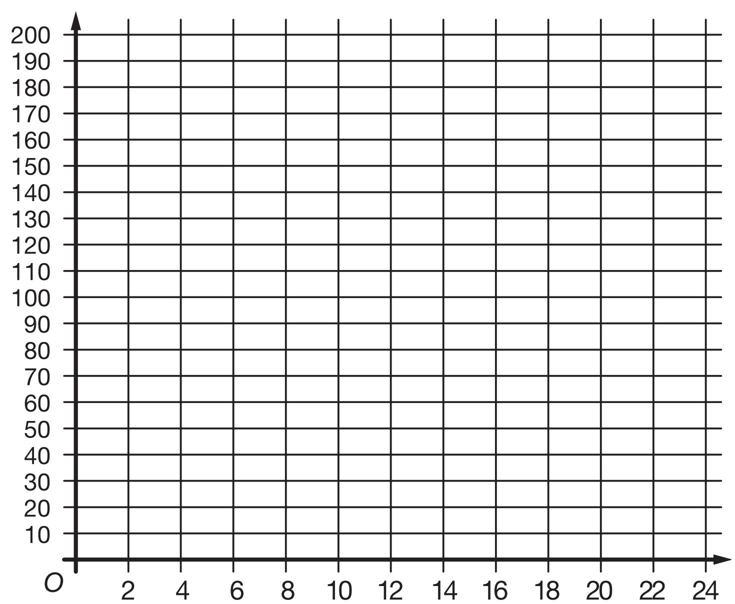
1. Define a variable for the quantity in Column 1. Enter this variable in the "Expression" row of the table under Column 1.

2. Write an expression that you can use to represent the quantity in Column 2 in terms of the quantity in Column 1. Enter this expression in the "Expression" row of the table under Column 2.

3. How long it will take you to bike 44 miles? Use the expression that you wrote in Question 2 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

4. How long it will take you to bike 164 miles? Use the expression that you wrote in Question 2 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

5. You will use the grid below to create a graph to represent the values in the table. Begin by labeling your axes. Use the first quantity from the table for the horizontal axis and the second quantity for the vertical axis. The axes are already numbered.



Investigate Problem 2

6. Write each row of numbers in your table as an ordered pair. Note that the *x*-coordinate is the amount of time you biked and the *y*-coordinate is the distance you traveled.

7. Plot each ordered pair on the prepared grid in Question 5.

8. What do you notice about the points you graphed on the coordinate plane? Write your answer using a complete sentence.

9. Draw a straight line through the points. How would you describe this line? Write your answer using a complete sentence.

10. Is the relation shown in the graph a linear function? Use a complete sentence to explain why or why not.

11. As the time increased, what happened to the distance? Use a complete sentence to explain.

12. **Math Path: Rate of Change**

As the time increased by 1 hour, by how much did the distance increase or decrease?

As well as describing the ratio of rise to run, slope can also be used to describe the rate of increase (or decrease) of the distance per 1 hour that you biked. This rate is a **rate of change** that can be described by slope when the units of measure on the *x*-axis and the *y*-axis are different. What is the slope of the line? Use the graph to help you. Write a complete sentence to explain your answer.

What units of measure are used for the slope in this problem?

Investigate Problem 2

13. Form a group with another partner team and compare your graphs and your answers to Questions 9 through 12. If you have any answers on which you do not agree, work together to find out why.

Problem 3 Funding the Trip

You have saved $100 for your bike trip. You are spending money at the rate of $12.50 per day.

A. How much money will you have left after 1 day? Use a complete sentence in your answer.

B. How much money will you have left after 3 days? Use a complete sentence in your answer.

C. How much money will you have left after 5 days? Use a complete sentence in your answer.

D. Which quantities are changing? Which quantities remain constant? Use complete sentences in your answers.

E. Which quantity depends on the other quantity? Use a complete sentence in your answer.

F. Complete the table below by first filling in the names of the quantities and their units of measure. Then fill in the table with your answers from Parts (A) through (C). The rest of the table will be completed later.

	Column 1	Column 2
Quantity Name		
Unit of Measure		
Problem 3, Part (A)	1	
Problem 3, Part (B)	3	
Problem 3, Part (C)	5	
Question 3		50
Question 4		0
Expression		

Investigate Problem 3

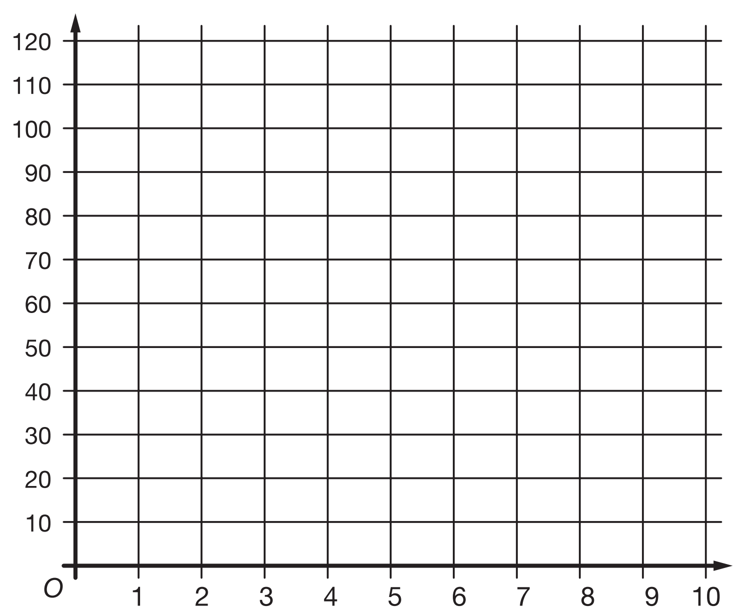
1. Define a variable for the quantity in Column 1. Enter this variable in the "Expression" row of the table under Column 1.

2. Write an expression that you can use to represent the quantity in Column 2 in terms of the quantity in Column 1. Enter this expression in the "Expression" row of the table under Column 2.

3. How long will it be until you have $50 left? Use the expression that you wrote in Question 2 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

4. How long will it be until you have no money left? Use the expression that you wrote in Question 2 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

5. You will use the grid below to create a graph to represent the values in the table. Begin by labeling your axes. Use the first quantity from the table for the horizontal axis and the second quantity for the vertical axis. The axes are already numbered.



Investigate Problem 3

6. Write each row of numbers in your table as an ordered pair. Note that the x-coordinate is the amount of time and the y-coordinate is the amount of money you have left.

7. Plot each ordered pair on the prepared grid in Question 5.

8. What do you notice about the points you graphed on the coordinate plane? Write your answer using a complete sentence.

9. Draw a straight line through the points. How would you describe this line? Write your answer using a complete sentence.

10. Is the relation shown in the graph a linear function? Use a complete sentence to explain why or why not.

11. As the time increased, what happened to the amount of money that you had left? Use a complete sentence to explain.

12. As the time increased by 1 day, by how much did the amount of money that you had left increase or decrease?

13. What is the slope of the line? Use the graph to help you. Write a complete sentence to explain your answer.

14. What units of measure are used for the slope in this problem?

15. Join your group with another and compare your graphs and your answers to Questions 9 through 14. If you have any answers on which you do not agree, work together to find out why. Be prepared to share your answers with the rest of the class.

13.4 Let's Have a Pool Party!

Finding Slope and y-Intercepts

Objectives

In this lesson, you will:

- Find *x*- and *y*-intercepts of a line.

Key Terms

- *x*-intercept
- *y*-intercept

We have used several different representations for linear functions including words, tables, equations, and graphs. In each of these representations, it is often important to be able to find two special points on the line—where the line crosses the *x*-axis and where the line crosses the *y*-axis.

Problem 1 Draining the Pool

Your friend wants to have a pool party, but she needs to drain and clean her pool first. The pool holds 6800 gallons of water, and can be drained at a rate of 20 gallons per minute.

A. How many gallons of water will be in the pool after 5 minutes? Use a complete sentence in your answer.

B. How many gallons of water will be in the pool after 30 minutes? Use a complete sentence in your answer.

C. How many gallons of water will be in the pool after 60 minutes? Use a complete sentence in your answer.

D. Which quantities are changing? Which quantities remain constant? Use complete sentences in your answers.

E. Which quantity depends on the other quantity? Use a complete sentence in your answer.

13

Investigate Problem 1

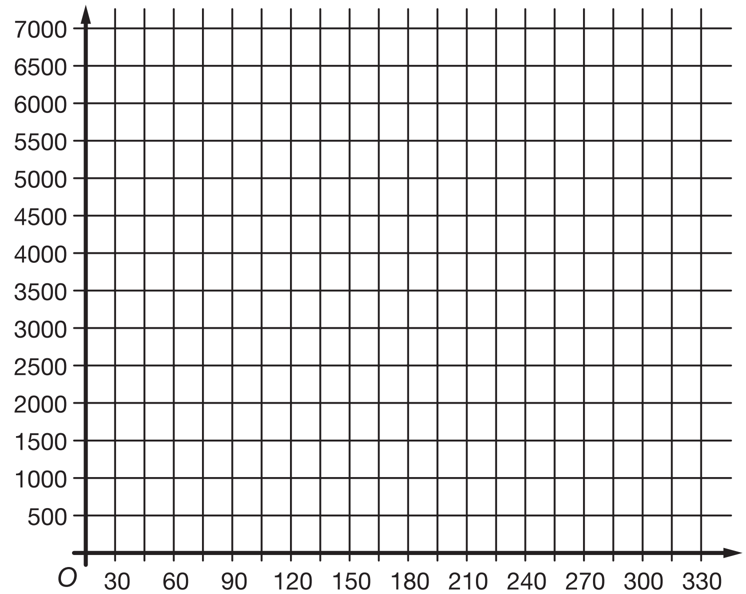
1. Complete the table below by first filling in the names of the quantities and their units of measure. Then fill in the table with your answers from Parts (A) through (C). The rest of the table will be completed later.

	Column 1	Column 2
Quantity Name		
Unit of Measure		
Problem 1, Part (A)	5	
Problem 1, Part (B)	30	
Problem 1, Part (C)	60	
Question 4		4200
Question 5		
Question 6		
Question 7		
Expression		

2. Define a variable for the quantity in Column 1. Enter this variable in the "Expression" row under Column 1.
3. Write an expression that you can use to represent the quantity in Column 2 in terms of the quantity in Column 1. Enter this expression in the "Expression" row under Column 2.
4. How long will it take until the pool has 4200 gallons of water left in it? Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table .
5. How long will it take until the pool is half full? Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.
6. How long will it take until the pool is one quarter full? Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.
7. How long will it take until there are only 10 more minutes left to drain the pool? (Hint: Determine the number of gallons that can be drained from the pool in 10 minutes.) Use the expression that you wrote in Question 3 to write an equation that you can solve to find your answer. Then write your answer in the appropriate place in the table.

Investigate Problem 1

8. You will use the grid below to create a graph to represent the values in the table. Begin by labeling your axes. Use the first quantity from the table for the horizontal axis and the second quantity for the vertical axis. The axes are already numbered.



9. Write each row of numbers in your table as an ordered pair. Note that the *x*-coordinate is the amount of time that the pool is draining and the *y*-coordinate is the number of gallons of water left in the pool.

10. Plot each ordered pair on the prepared grid in Question 8.

11. Draw a straight line through the points. Be sure to draw the line so that it intersects both the *x*-axis and the *y*-axis.

12. Is the relation shown in the graph a linear function? Use a complete sentence to explain why or why not.

13. As the time increased, what happened to the amount of water in the pool? Use a complete sentence to explain.

Investigate Problem 1

14. As the time increased by 1 minute, by how much did the amount of water in the pool increase or decrease? Use a complete sentence to explain.

15. What is the slope of the line? Use the graph to help you. Write a complete sentence to explain your answer.

16. What units of measure are used for the slope in this problem?

17. **Math Path: *x*- and *y*-Intercepts**

The ***x*-intercept** of a graph is the *x*-coordinate of the point where the graph crosses the *x*-axis. The ***y*-intercept** of a graph is the *y*-coordinate of the point where the graph crosses the *y*-axis.

Use your graph to find the point where the graph crosses the *x*-axis. Write the point as an ordered pair. What does this point represent in the problem? Use a complete sentence in your answer.

What is the *x*-intercept?

Use your graph to find the point where the graph crosses the *y*-axis. Write the point as an ordered pair. What does this point represent in the problem? Use a complete sentence in your answer.

What is the *y*-intercept?

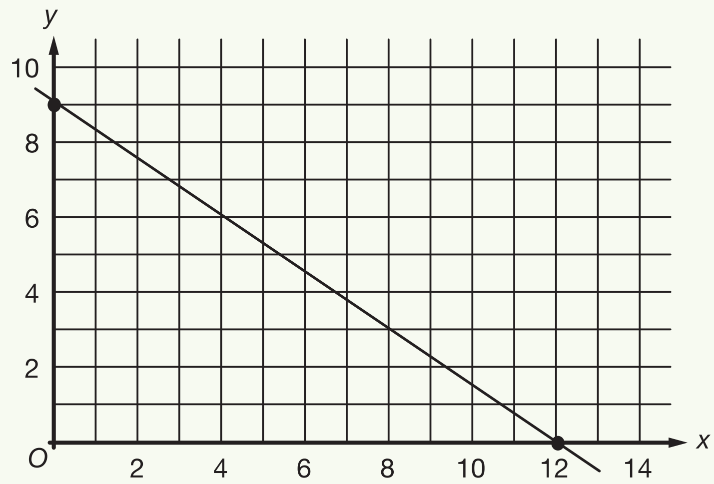
Use complete sentences to explain what you notice about the ordered pairs that include the *x*- and *y*-intercepts.

18. Form a group with another partner team and compare your graphs and your answers to Questions 12 through 17. If you have any answers on which you do not agree, work together to find out why. Be prepared to share your work with the rest of the class.

Problem 2 *Identifying Slopes and Intercepts*

For each representation of a linear function below, find the slope and *x*- and *y*-intercepts.

A.



Slope = ____________ (Hint: the slope in this case is negative.)

x-intercept = ____________

y-intercept = ____________

B.

x	*y*
0	0
1	4
3	12
4	16

Write each row of numbers in the table as an ordered pair.

You can use the coordinates of two points to find the slope of the line through the points. Let the point (x_1, y_1) be the point (1, 4) and (x_2, y_2) be the point (3, 12). Complete the ratio below to find the slope of the line through the points.

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square}$$

Slope = ____________

x-intercept = ____________

y-intercept = ____________

13.5 What's for Lunch?

Using Slope and Intercepts to Graph Lines

Objectives

In this lesson, you will:

- Graph lines using slopes and intercepts.
- Use the slope-intercept form of the equation of a line.

Key Terms

- linear equation
- slope-intercept form

Now that we can find the slope and intercepts of a linear function, we can use them in order to graph linear functions more easily.

Problem 1 Lunch Plan A

Your school has three different plans for buying school lunches. In Plan A, you pay \$2 per day for a hot lunch. A linear function that models the total amount that you pay for Plan A is $y = 2x$, where x is the number of days that you buy lunch.

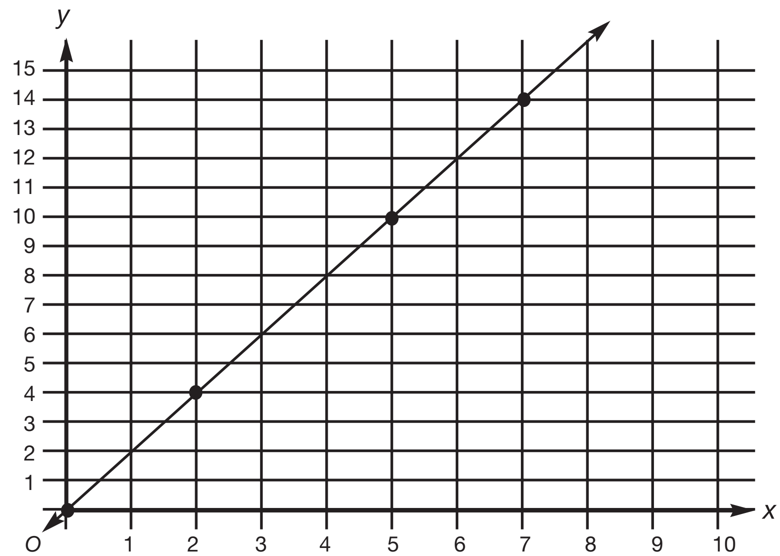
A. The slope of the linear function is 2. This means that every time the value of the independent variable increases by 1 unit, the value of the dependent variable increases by 2 units. What happens to the value of the dependent variable if the value of the independent variable *decreases* by 1 unit? Write your answer using a complete sentence.

B. By how many units would the value of the dependent variable increase or decrease if the value of the independent variable *increases* by 4 units? Write your answer using a complete sentence.

C. By how many units would the value of the dependent variable increase or decrease if the value of the independent variable *decreases* by 4 units? Write your answer using a complete sentence.

Investigate Problem 1

1. Below is the graph of the linear function $y = 2x$.



What are the x- and y-intercepts? Write your answer using a complete sentence.

2. Place your pencil on the point on the line where $x = 2$. Move your pencil one unit to the right. This represents an increase of the independent variable by one unit. By how many units up or down do you need to move your pencil to get back to the line? Write your answer using a complete sentence.

3. Repeat the process in Question 2, starting at the point on the line where $x = 6$. Use a complete sentence to describe what you notice.

4. Repeat the process in Question 2, starting at another point on the line. What can you conclude? Write your answer using a complete sentence.

13

Problem 2 Lunch Plan B

In your school's lunch Plan B, you pay a weekly fee of \$10 that goes into your account. When you buy lunch, \$2 per day is subtracted from your account balance. A linear function that models the total amount that you have in your account in Plan B is $y = 10 - 2x$, where x is the number of days that you buy lunch.

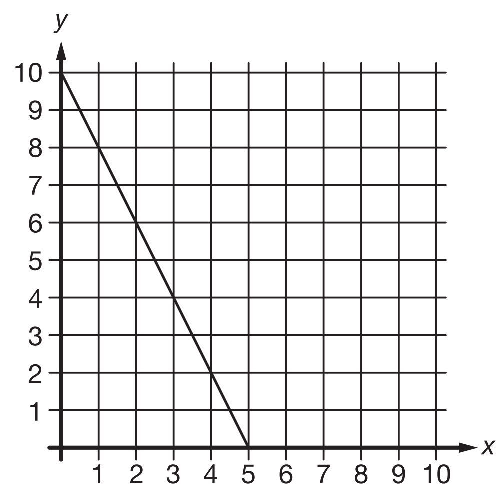
A. The slope of the linear function is –2. This means that every time the value of the independent variable increases by 1 unit, the value of the dependent variable decreases by 2 units. What happens to the value of the dependent variable if the value of the independent variable *decreases* by 1 unit? Write your answer using a complete sentence.

B. By how many units would the value of the dependent variable increase or decrease if the value of the independent variable *increases* by 4 units?

C. By how many units would the value of the dependent variable increase or decrease if the value of the independent variable *decreases* by 4 units?

Investigate Problem 2

1. Below is the graph of the linear function $y = 10 - 2x$.



What are the x- and y-intercepts? Write your answer using a complete sentence.

13

Investigate Problem 2

2. Place your pencil on the point on the line where $x = 2$. Move your pencil one unit to the right. This represents an increase of the independent variable by one unit. By how many units up or down do you need to move your pencil to get back to the line? Write your answer using a complete sentence.

3. Repeat the process in Question 2, starting at the point on the line where $x = 4$. Use a complete sentence to describe what you notice.

4. Repeat the process in Question 2, starting at another point on the line. What can you conclude? Write your answer using a complete sentence.

5. Complete the table to find the money in your account each week for lunch Plan B when buying lunch for different numbers of days.

x	*y*
0	
1	
3	
5	

6. **Math Path: Linear Equation**

   Write each row of numbers in the table as an ordered pair. These ordered pairs are said to be solutions of the *linear equation* $y = 10 - 2x$. In a **linear equation** in two variables, the variables are raised to the first power (such as t, not t^2) and appear only once.

Take Note

Although all linear equations are functions, not every equation is a function. You will discover this in a later math course.

7. Compare the *y*-intercept that you found in Question 1 and the slope from Part (A) with the equation $y = 10 - 2x$. What do you notice? Write your answer using a complete sentence.

Investigate Problem 2

13

8. Math Path: Slope-Intercept Form

When you graph the equation $y = 10 - 2x$, the graph is a straight line. This equation can also be written as $y = -2x + 10$. When an equation is written in this form, it is said to be in *slope-intercept form.* When a linear equation is written in the form $y = mx + b$, it is in **slope-intercept form.**

What do you think the variable m represents in the equation $y = mx + b$? Write your answer using a complete sentence.

What do you think the variable b represents in the equation $y = mx + b$? Write your answer using a complete sentence.

9. Use a complete sentence to explain why you think that the form $y = mx + b$ is called the slope-intercept form.

10. What is the slope of a line whose equation is $y = -3x + 24$? Write your answer using a complete sentence.

11. What is the y-intercept of a line whose equation is $y = -3x + 24$? Write your answer using a complete sentence.

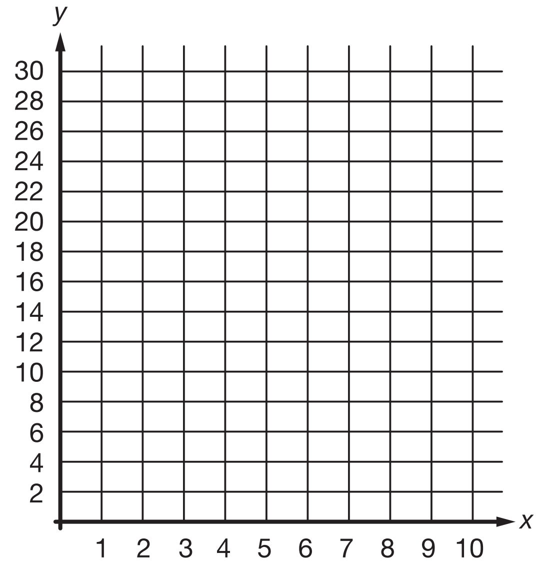
12. Evaluate the expression to complete the table.

	x	y
Expression	x	$-3x + 24$
	1	
	2	
	3	
	4	
	5	
	6	

13

Investigate Problem 2

13. Write each row of numbers in the table as an ordered pair. Then plot each ordered pair below.



14. Find the slope and the *y*-intercept from the graph. Then compare these with the slope and the *y*-intercept from Questions 10 and 11. Write your findings using a complete sentence.

15. You can also find the *x*-intercept of a line by substituting a 0 into the equation for *y* and solving for *x*. You can find the *y*-intercept in a similar way—substitute a 0 into the equation for *x* and solve for *y*. Use this method to find the *x*- and *y*-intercepts for the graph of the equation $y = 3x - 6$ by completing the boxes.

x-intercept:

$y = 3x - 6$

$0 = 3x - 6$

$\square = 3x$

$\square = x$

y-intercept:

$y = 3x - 6$

$y = 3(0) - 6$

$y = 0 - \boxed{\bullet}$

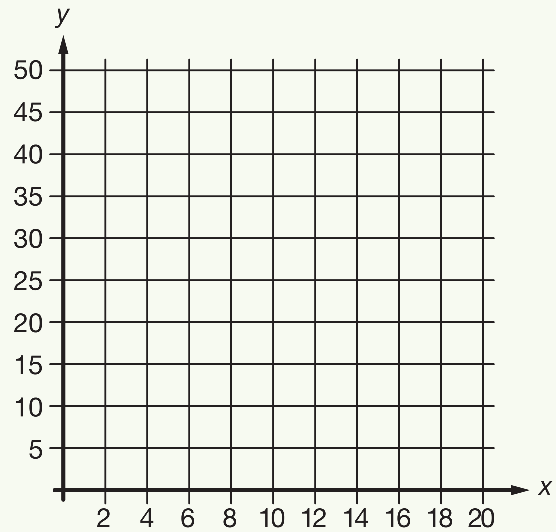
$y = \square$

Use this method to find the *x*- and *y*-intercepts for the graph of the equation $y = -7x + 14$.

Problem 3 Lunch Plan C

In your school's lunch Plan C, you pay a monthly fee of $25 that goes into your account. When you buy lunch, you pay only $.50 per day. A linear function that models the total amount that you pay for Plan C is $y = 25 + 0.5x$, where x is the number of days that you buy lunch.

A. On the grid below, graph the y-intercept of the graph of the equation $y = 25 + 0.5x$.



B. The slope of the line is 0.5. What happens to the value of y when x increases by 1 unit? Write your answer using a complete sentence.

C. Place your pencil on the y-intercept. Move your pencil two units to the right. Then based on your slope, move your pencil the correct number of units up or down. What are the coordinates of the point that you land on? Draw the point.

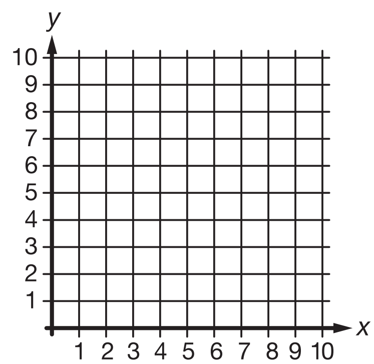
D. Repeat the process in Part (C) two more times to draw two additional points. What are the coordinates of the points?

Draw a line through your points and the y-intercept.

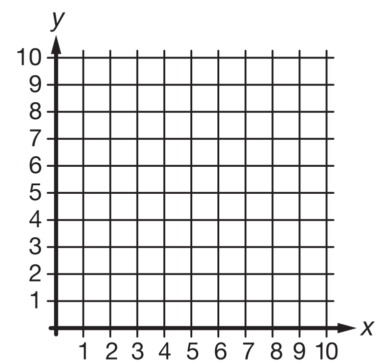
Investigate Problem 3

Use the method outlined in Problem 3 to graph each linear equation written in slope-intercept form.

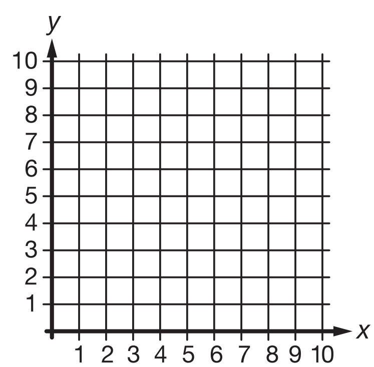
$y = 3x + 5$



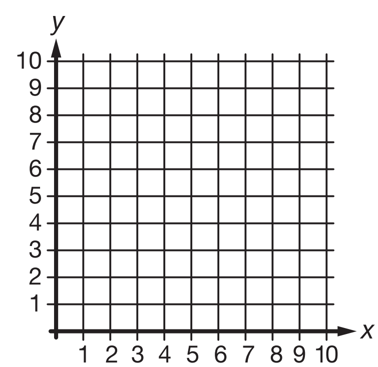
$y = 5x$



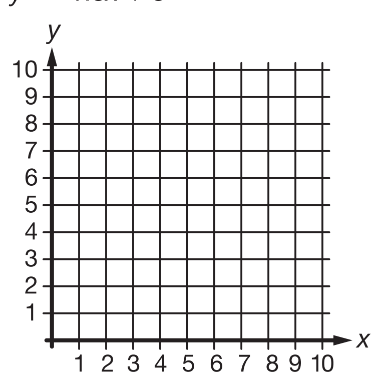
$y = -4x + 7$



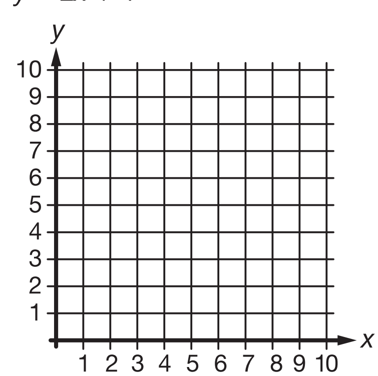
$y = 2.5x$



$y = -1.5x + 5$



$y = 2x + 4$



13.6 Healthy Relationships

Finding Lines of Best Fit

Objectives

In this lesson, you will:

- Create scatter plots of data.
- Find a line of best fit for a set of data.

Key Terms

- scatter plot
- line of best fit

Linear functions are very useful models of many everyday situations, as we have seen in the last few lessons. They can also be used to model sets of data approximately, when the points do not all lie on a straight line.

Problem 1 The Leonardo Da Vinci Problem

Leonardo Da Vinci was a famous artist and mathematician who discovered a unique relationship between the height of a person and the span of the person's arms.

A. Your teacher has taped two meter sticks together for each group. Use the taped meter sticks to measure each person's height and the distance between the tips of the person's longest fingers with his or her arms spread wide. The second measurement is called a person's arm span.

Record the measurements to the nearest centimeter in the table.

Group Member's Name	Height (centimeters)	Arm Span (centimeters)

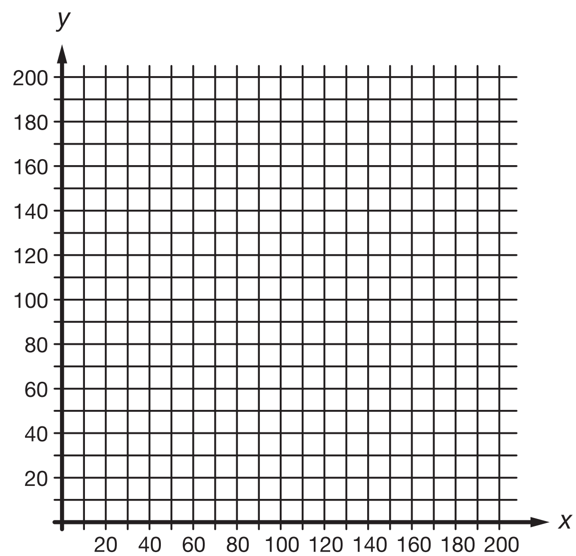
B. Write each row of numbers in the table as an ordered pair. Note that the *x*-coordinate is the height and the *y*-coordinate is the arm span.

C. Each group should share their data with the entire class. Take turns writing your group's ordered pairs so that the class can see them.

13

Investigate Problem 1

1. Use the grid below to create a graph of all of the ordered pairs collected in your class. Begin by labeling the horizontal axis as "Height (centimeters)" and the vertical axis as "Arm Span (centimeters)." The axes are already numbered. The graph is called a *scatter plot.* A **scatter plot** is a graph that shows how two sets of data are related.



2. Use a complete sentence to describe the shape of the graph of all of the data points.

3. Does the pattern of the points taken as a group increase or decrease as you move from left to right? Write your answer using a complete sentence.

4. As a group, decide where to draw a line through the points that would best "fit" the data, called the *line of best fit.* The **line of best fit** is the line that is very close to most of the points. Use a ruler to draw the line. Be sure to extend the line so that it intersects the x-axis and the y-axis.

5. What is the y-intercept of your "line of best fit?" Write your answer using a complete sentence.

Investigate Problem 1

6. Beginning at the *y*-intercept, move one unit to the right. How many units must you go up or down to get back to the line? Write your answer using a complete sentence.

7. What does the ratio of the number in Question 6 to the number 1 represent? Write your answer using a complete sentence.

8. Using the ratio in Question 7 and the *y*-intercept, write an equation of the line of best fit.

9. Using the equation you wrote in Question 8 to answer the following questions.

   If a person's height is 100 centimeters, what is the person's arm span?

   If a person's height is 50 centimeters, what is the person's arm span?

   If a person's height is 75 centimeters, what is the person's arm span?

   If a person's arm span is 100 centimeters, what is the person's height?

10. Write each answer to Question 9 as an ordered pair. Remember that the *x*-coordinate is the height and the *y*-coordinate is the arm span.

11. Add the ordered pairs from Question 10 to your graph using a pen or pencil of a different color. How well do these points fit in with the original data set? Write your answer using a complete sentence.

12. Do you think that your line is a "good" model of the data? Use a complete sentence to explain your reasoning.

13. What are the advantages of having a model, that is, having an equation of the line of best fit, for this set of data? Write your answer using a complete sentence.

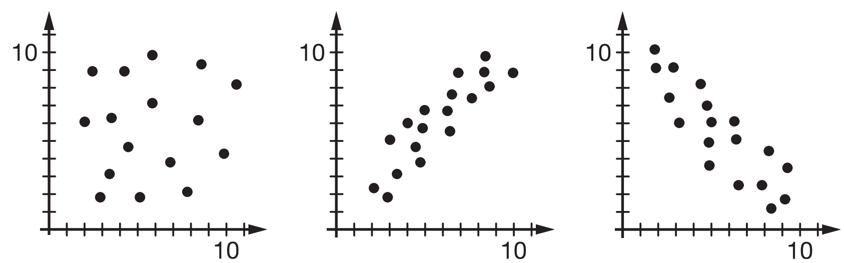
Take Note

The data in a scatter plot are said to have a positive relationship when the line that models the data has positive slope, a negative relationship when the line that models the data has negative slope, and no relationship when a line should not be used to model the data.

13

Investigate Problem 1

14. In a scatter plot, the two sets of data that are graphed can have a positive relationship, a negative relationship, or no relationship. Under each graph below, write either “positive relationship,” “negative relationship,” or “no relationship.”



Problem 2 Children’s Proportions

The ratio of the height of a child’s head (from the chin to the top of the head) to the child’s total height changes as the child grows older. For instance, the ratio of an infant’s head height to his or her total height is about 1 to 3. The ratio of an adult’s head height to his or her total height is about 1 to 7.

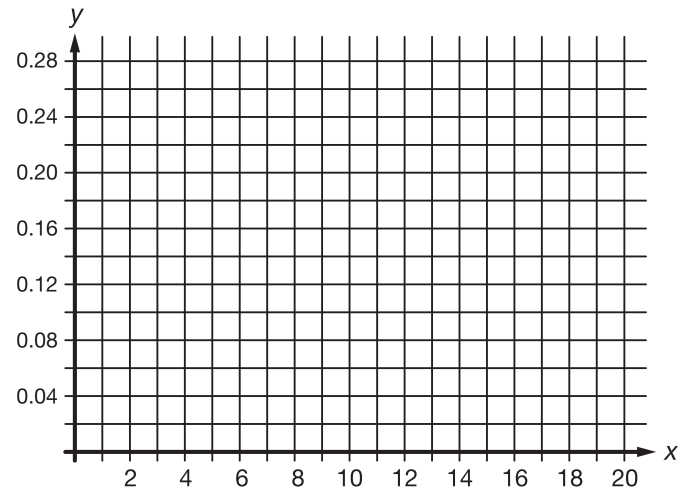
For a project for health class, you want to prove that this is true, so you measure the total height and the head height of 10 children of different ages in your neighborhood. The table lists the data that you collect.

Age of Child (years)	Head Height (centimeters)	Total Height (centimeters)	Ratio of Head Height to Total Height
12	22.8	152	0.15
2	19.8	90	0.22
14	23.1	165	0.14
13	22.2	148	0.15
5	22.8	114	0.20
10	22.4	140	0.16
3	20.2	101	0.20
6	20.9	110	0.19
9	21.6	135	0.16
7	18.0	100	0.18

At the left, write each row of numbers in the table as an ordered pair. Note that the *x*-coordinate is the age and the *y*-coordinate is the ratio of the head height to the total height.

Investigate Problem 2

1. Use the grid below to create a graph of all of the ordered pairs in Problem 2. Begin by labeling the horizontal axis as "Age (years)" and the vertical axis as "Ratio of Head Height to Total Height." The axes are already numbered.



2. Use a complete sentence to describe the shape of the graph of all of the data points.

3. Does the pattern of the points taken as a group increase or decrease as you move from left to right? Write your answer using a complete sentence.

4. With your partner, decide where to draw a line through the points so that it is the line of best fit. Then use a ruler to draw the line. Be sure to extend the line so that it intersects the *y*-axis.

5. What is the *y*-intercept of your "line of best fit"? Write your answer using a complete sentence.

13

13

Investigate Problem 2

6. Beginning at the y-intercept, move one unit to the right. How many units must you go up or down to get back to the line? Write your answer using a complete sentence.

7. What does the ratio of the number in Question 6 to the number 1 represent? Write your answer using a complete sentence.

8. Using the ratio in Question 7 and the y-intercept, write an equation of the line of best fit.

9. Use the equation you wrote in Question 8 to answer the following questions.

If a little girl is 4 years old, what is the ratio of the girl's head height to her total height?

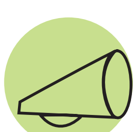
If a boy is 8 years old, what is the ratio of the boy's head height to his total height?

If a girl is 16 years old, what is the ratio of the girl's head height to her total height?

If the ratio of a boy's head height to his total height is 0.13, how old is the boy?

10. Write each answer to Question 9 as an ordered pair. Remember that the x-coordinate is the age and the y-coordinate is the ratio of the head height to the total height.

11. Add the ordered pairs from Question 10 to your graph using a pen or pencil of a different color. How well do these points fit in with the original data set? Write your answer using a complete sentence.



12. Do you think that your line is a "good" model of the data? Use a complete sentence to explain your reasoning.

Looking Back at Chapter 13

Key Terms

- relation ● p. 419
- input ● p. 419
- output ● p.419
- function ● p. 419
- input-output table ● p. 420
- independent variable ● p. 421
- dependent variable ● p. 421
- domain ● p. 422
- range ● p. 422
- function notation ● p. 422
- linear function ● p. 423
- rise ● p. 427
- run ● p. 427
- slope ● p. 427
- rate of change ● p. 430
- *x*-intercept ● p. 438
- *y*-intercept ● p. 438
- linear equation ● p. 444
- slope-intercept form ● p. 445
- scatter plot ● p. 450
- line of best fit ● p. 450

Summary

Determining Whether Relations are Functions (p. 421)

A function is a special relation in which every *x*-coordinate is paired with one and only one *y*-coordinate.

Examples

The relation represented by the ordered pairs (1, –1), (2, 3), (1, –3), (–4, 2) is not a function because the *x*-coordinate 1 is paired with the *y*-coordinates –1 and –3.

The relation represented by the ordered pairs (–2, 4), (–1, 1), (1, 1), (2, 4) is a function because all of the *x*-coordinates are paired with one and only one *y*-coordinate.

Using Function Notation (p. 422)

To write an equation using function notation, replace the dependent variable *y* with the notation *f*(*x*), which is read as "*f* of *x*" or "the function of *f* at *x*."

Examples

Original equation:	$y = 3 + x$	$y = 4x - 10$	$y = 6 - (x + 2)$
Function notation:	$f(x) = 3 + x$	$f(x) = 4x - 10$	$f(x) = 6 - (x + 2)$

Finding Slope (p. 427)

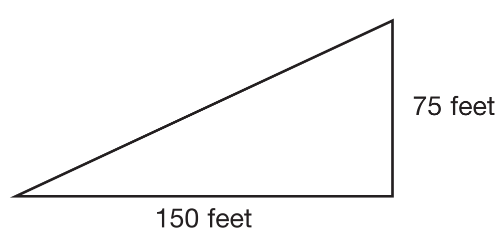
To find the slope as a ratio, divide the rise, or vertical change, by the run, or horizontal change.

Example

To find the slope of the ski hill, divide the change in vertical distance by the change in horizontal distance.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{75 \text{ feet}}{150 \text{ feet}}$$



So, the slope of the ski hill is $\frac{75}{100}$, or $\frac{1}{2}$.

Using Slope to Describe a Rate of Change (p. 430)

To use slope to describe a rate of change, divide the amount of change in the *y*-coordinates by the amount of change in the *x*-coordinates. The *x*-coordinates and the *y*-coordinates will have different units.

13

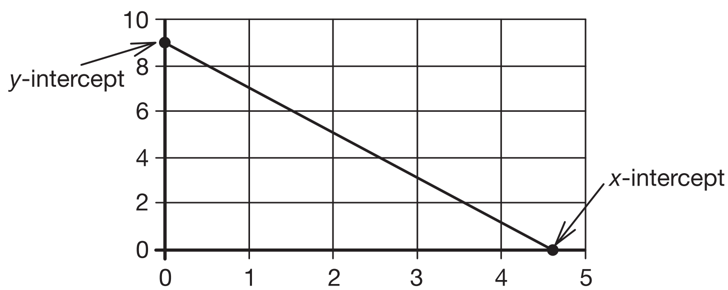
Example Linda swam 400 meters in 8 minutes.

$$\text{Slope} = \frac{400 \text{ meters}}{8 \text{ minutes}} = 50$$

Finding x- and y-Intercepts (p. 438)

The *x*-intercept is the point where the graph crosses the *x*-axis and the *y*-intercept is the point where the graph crosses the *y*-axis.

Example The graph of $y = -2x + 9$ is shown below.



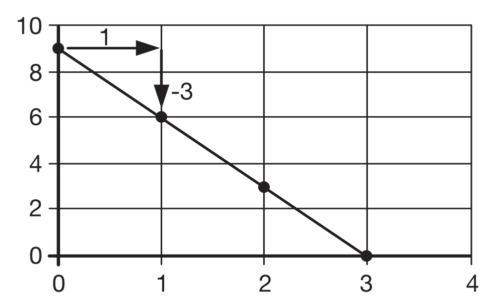
The graph crosses the *x*-axis at $\left(\frac{9}{2}, 0\right)$. The graph crosses the *y*-axis at (0, 9).
So, the *x*-intercept is $\frac{9}{2}$ and the *y*-intercept is 9.

Graphing Lines Using Slopes and Intercepts (p. 444)

To graph a linear equation, write the linear equation in slope-intercept form. Identify the slope and graph the *y*-intercept. Place your pencil on the *y*-intercept. Then choose a value for *x* and move that many units to the left or right. Based on the slope, move your pencil the correct number of units up or down and draw a point. Repeat this process until you have enough points to draw a straight line.

Example For the linear equation $y = -3x + 9$, the slope is –3 and the *y*-intercept is 9.

Place your pencil on the *y*-intercept 9. Then choose the value $x = 1$ and move 1 unit to the right and 3 units down because the slope is –3.



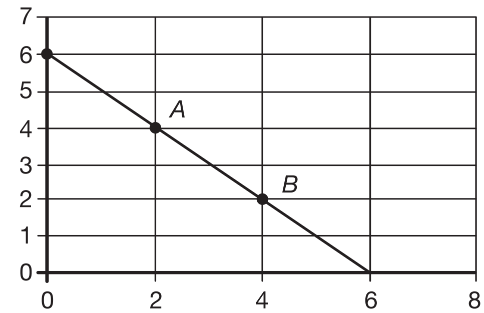
13

Finding the Equation of a Line from a Graph (p. 445)

To find the equation of a line from its graph, first identify the y-intercept. Then choose two convenient points on the line. To find the slope, use the two points in the slope equation. Then use the slope and y-intercept to write the equation of the line in slope-intercept form.

Example

From the graph, you can see that the y-intercept is 6. Choose Point A (2, 4) as (x_1, y_1) and Point B (4, 2) as (x_2, y_2).



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{4 - 2} = \frac{-2}{2} = -1$$

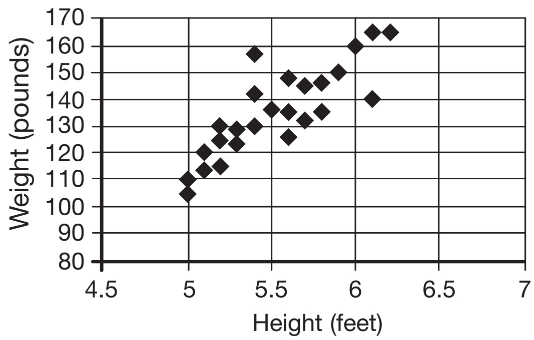
So, the equation of the line is $y = -1x + 6 = -x + 6$.

Creating Scatter Plots of Data (p. 450)

To create a scatter plot of data, graph the data as ordered pairs.

Example

The relationship between height and weight of women is shown below as a scatter plot.



Finding a Line of Best Fit (p. 450)

To find a line of best fit for a set of data, draw a straight line that is very close to most of the points. Be sure that the line intersects the x-axis and the y-axis.

Example

The line of best fit for the data that shows the relationship between height and weight of women is shown in the graph.