Looking Ahead to Chapter 12

Focus In Chapter 12, you will work with volume and surface area of three-dimensional figures. You will learn about prisms, pyramids, cones, cylinders, and spheres. You will also design nets and construct top, side, and front views of three-dimensional objects.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 12.

Find the product.

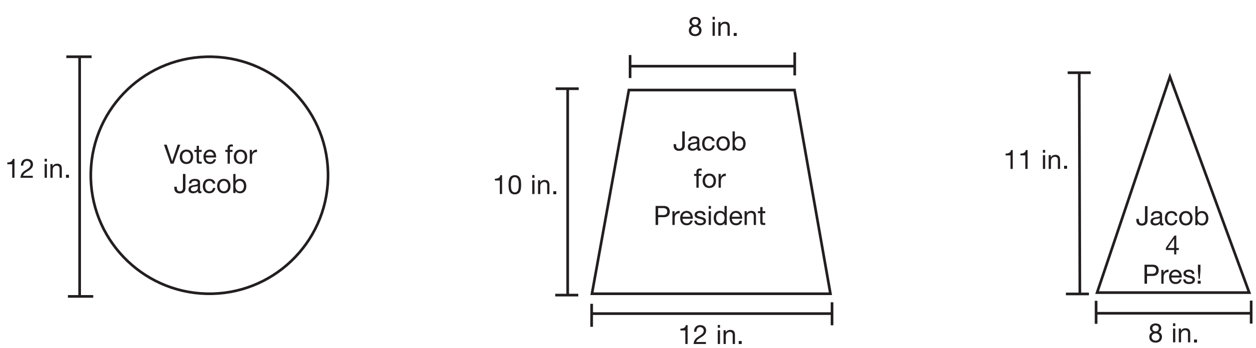
1. $3 \times 4 \times 2.5$
2. $6.5 \times 9 \times 9$
3. $11 \times 2 \times 5$

Evaluate the expression. Use 3.14 for π.

4. 30π
5. $2\pi(15)$
6. $\pi(4)^2$

Read the problem scenario below.

Jacob is running for class president and is making posters for his campaign. He wants to make the posters in different shapes. Below are three posters that Jacob made.



7. Write the name of the shape of each poster below it.
8. What is the area of each poster?
9. How many square inches of paper did Jacob use to make all three posters?

Key Terms

12

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Volume and Surface Area

There are in excess of 10,000 different packages and containers to choose from in the average supermarket, including both food and non-food items. In Lesson 12.4, you will design containers that have the greatest volume and use the least amount of material.

12.1

Your Friendly Neighborhood Grocer

Three-Dimensional Figures

Objectives

In this lesson, you will:

- Identify three-dimensional figures.

Key Terms

- prism
- face
- base
- height
- vertex
- edge
- solid
- polyhedron
- pyramid
- cylinder
- cone
- sphere

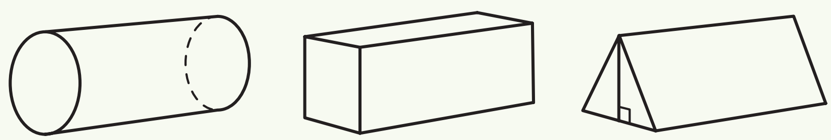
Take Note

The prism in the middle of the figures above is called a *rectangular* prism.

In Chapters 9 and 10, you studied two-dimensional figures. These two-dimensional figures are used to form figures in three dimensions.

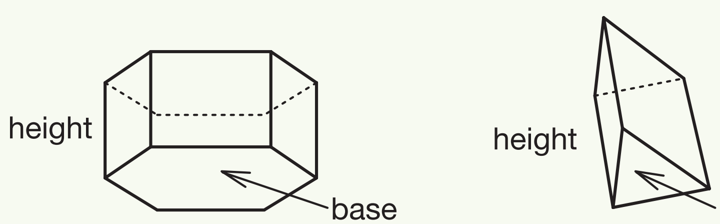
Problem 1 Packing Meat

Suppose that a butcher in a grocery store needs to package ground meat. He needs to decide between the three packages below.



A. Which package would stack better in the display case and why? Which package would roll better in a packaging machine and why? Write your answers using complete sentences.

B. The figures above in the center and on the right are *prisms.* A **prism** is a three-dimensional figure consisting of flat surfaces called **faces.** Two of the faces are parallel and congruent polygons called **bases.** The other faces are rectangles. The **height** of a prism is the length of a segment that is perpendicular to the bases and joins the bases. A prism is named for the shape of its bases.

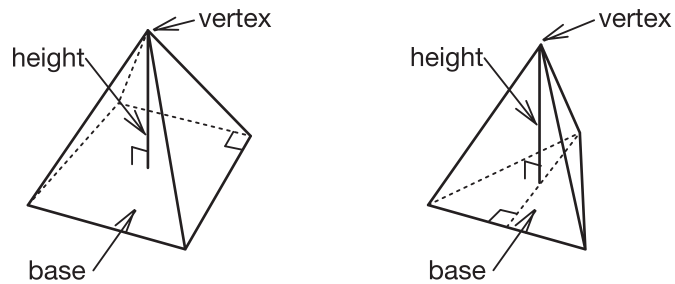


hexagona: prism triangular prism

Mark an X on each face of the hexagonal and triangular prisms. A **vertex** of a prism is the point where three faces of the prism intersect. Draw a dot on each vertex. An **edge** is where two faces of a prism intersect in a line. Draw a line on each edge.

Investigate Problem 1

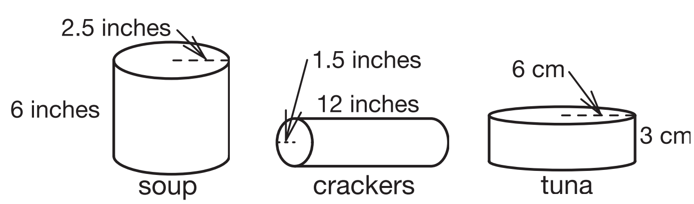
1. The prisms in Problem 1 are part of a group of **solids,** or three-dimensional figures, called *polyhedrons.* A **polyhedron** is a solid that has faces that are polygons. Another type of polyhedron is a *pyramid.* A **pyramid** is a solid that has triangular faces that meet at one point, a common vertex. The base of a pyramid is a polygon. A pyramid is named for the shape of its base. The height of a pyramid is the length of the segment from the common vertex to the base such that the segment is perpendicular to the base.



________________ pyramid ________________ pyramid

Decide whether each pyramid above is a rectangular pyramid or a triangular pyramid. Write your answers on the lines. Then use complete sentences to explain your answers.

2. One of the butcher's possible packages in Problem 1 was not a polyhedron but a *cylinder.* A **cylinder** is a solid that has two congruent parallel bases that are circles. Many groceries are packaged in cylinders. The height of the cylinder is the length of a segment that is perpendicular to the bases and joins the bases.



What is the height of each cylinder?

What is the radius of each cylinder?

What is the diameter of each cylinder?

Investigate Problem 1

3. Two other solids that are not polyhedrons are a *cone* and a *sphere*. A **cone** is a solid that has one circular base and one vertex. A **sphere** is the set of all points in space that are the same distance from a center point. Identify each item below as a prism, cone, sphere, or cylinder.

4. Explain to your partner the similarities and differences between cones and pyramids.

5. Identify buildings or other objects in your environment that are shaped like these five geometric solids—prisms, pyramids, cylinders, cones, and spheres. Keep an ongoing list of your findings in the margin at the left.

6. **Math Path: Euler's Formula**

   In 1752, Swiss mathematician Leonhard Euler (pronounced "oiler") stated a formula that related the number of faces, vertices, and edges of geometric solids whose faces are polygons. Use toothpicks and clay to build each polyhedron. Then complete the table.

		Solid	Number of Edges, *E*	Number of Faces, *F*	Number of Vertices, *V*
Polyhedrons	Prisms	triangular			
		rectangular			
		pentagonal			
		hexagonal			
	Pyramids	triangular			
		rectangular			
		pentagonal			
		hexagonal			

   Do you see a pattern or relationship? State your pattern or relationship using words or variables.

   Do you think this relationship works for all pyramids and prisms? Discuss this with your group.

Take Note

Remember that a *face* of a polyhedron is any flat surface of the polyhedron, a *vertex* is a point where three faces of a polyhedron intersect, and an *edge* is the line formed when two faces of a polyhedron intersect.

12.2 Carnegie Candy Company

Volumes and Surface Areas of Prisms

Objectives

In this lesson, you will:

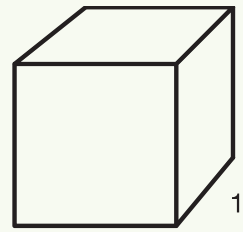
- Find volumes of rectangular and triangular prisms.
- Find surface areas of rectangular and triangular prisms.

Key Terms

- cube
- volume
- surface area

Problem 1 Candy Cubes

The Carnegie Candy Company decides to make mathematical candy packages. The company makes caramels in the shape of a *cube*. A **cube** is a special prism with faces that are all congruent squares, as shown at the right. The caramel cubes are 1 inch by 1 inch by 1 inch.



A. Although most candy companies package their candies in bags or long rectangular boxes, the Carnegie Candy Company packages their caramels in cubic packages. Because of this, they can only make packages of certain sizes. What sizes can they make, and how many caramels will be in each package? Complete the table.

Size of Package (cubic inches)	Number of Caramels
$1 \times 1 \times 1$	1
$2 \times 2 \times 2$	8
$3 \times 3 \times 3$	
$4 \times 4 \times 4$	
$5 \times 5 \times 5$	
⋮	⋮
$s \times s \times s$	

B. A package contains 216 caramels. How are the caramels arranged in the package? Use a complete sentence to explain.

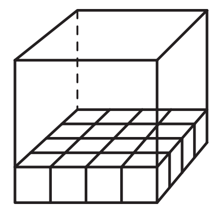
C. A package contains 7^3 caramels. How many caramels are layered on each face? Write your answer using a complete sentence.

Investigate Problem 1

1. Math Path: Volume

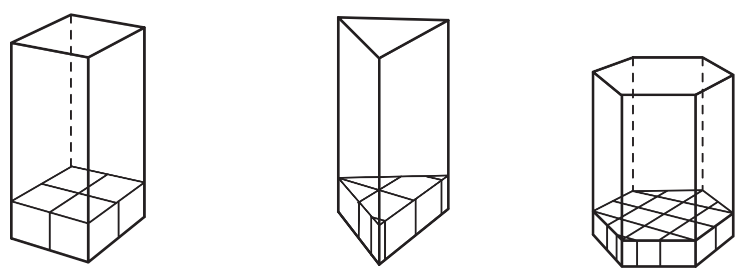
The **volume** of a solid is the amount of space occupied by the solid, or the capacity of the solid. You can think of volume as the number of unit cubes that will fit into a three-dimensional shape. A unit cube has a length, width, and height of 1 unit and is the basic unit of measurement for volume.

One strategy for finding the volume of a rectangular prism is to count the number of layers of unit cubes it takes to fill the rectangular prism. The number of unit cubes in a layer is the same as the area of the base. So, the number of unit cubes in a rectangular prism (the volume) is the area of the base multiplied by its height. Find the volume of the cube at the right.



In all prisms, the volume is the area of the base B multiplied by the height h. This is written mathematically as

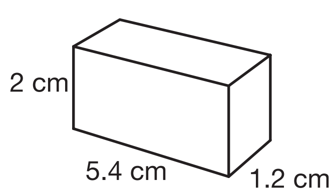
Volume of a prism = $B \times h$.



If the prism is a rectangular prism, then the area of the base is equal to the length of the base times the width of the base and the formula becomes

Volume of a rectangular prism = $\quad \times w \times h$.

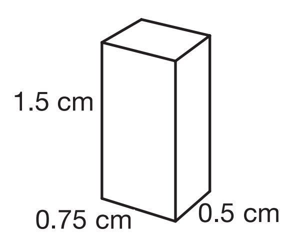
2. Find the volume of each rectangular prism.



$V = \quad \times w \times h$

= ____ × ____ × ____

= ____ cubic centimeters



$V = \quad \times w \times h$

= ____ × ____ × ____

= ____ cubic inches

Take Note

When you are using measurements in inches, feet, or yards, the volume is measured in cubic inches (or inches^3), cubic feet (or feet^3), or cubic yards (or yards^3). In the metric system, if you are measuring in centimeters or meters, then volume is measured in cubic centimeters (or centimeters^3) or cubic meters (or meters^3). This makes sense mathematically because three-dimensional figures have a length, a width, and a height.

Problem 2 Cheaper By the Dozen

The Carnegie Candy Company has a special order for 2 dozen caramels packaged together in a box that is a rectangular prism. They have a problem, though. The company always packages their caramels in cubes and 24 is not the cube of an integer. They need to find all of the possible dimensions of a box that would hold exactly 24 cubic caramels.

A. Use sugar cubes or some other type of cube to find all of the possible arrangements of 24 cubes.

B. Complete the table. Note that you may or may not need all of the rows. Do not use the last column yet. This will be completed later.

Length	Width	Height	Volume (cubic units)	

How many different arrangements did you find?

Were all of the volumes of the boxes equal to 24?

C. Because the designers are designing a new box that they have never used before, the box needs to be as inexpensive as possible so that they can make a profit. Because the material used to make the boxes is very expensive (bright colored foil), the company needs to be sure that they make the *surface area* of the box as small as possible. The **surface area** is the sum of all of the areas that form the surface of the solid. In this case, the surface area is the area of all six faces—top, bottom, left side, right side, front, and back.

Investigate Problem 2

1. **Math Path: Surface Area**

   Label the last column of your table “Surface Area (square units).” Surface area is measured in square units because each face is a two-dimensional figure.

   To find the surface area, you can count the square units on each face of your box. Or, you can sketch each box and find the area of each face and then add the areas. Complete the table.

2. Which box in the table had the smallest surface area? How would you describe its shape? Use a complete sentence to explain why it had the smallest surface area.

3. Write the formula for finding the volume of any rectangular prism.

4. Write the formula for finding the surface area of any rectangular prism.

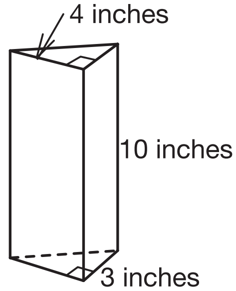
5. Using what you decided as the best way to package 24 caramels, how would you package 36 caramels to make the surface area of the box as small as possible? Explain your answer using a complete sentence.

6. How would you package 48 caramels to make the surface area of the box as small as possible? Explain your answer using a complete sentence.

7. On March 14 (Pi Day), the Carnegie Company wants to package the caramels in a box that has the same volume and surface area. Is this possible? Use a complete sentence to explain your thinking mathematically.

Investigate Problem 2

8. The Carnegie Candy Company is deciding whether to begin selling chocolate milk at their store in containers that are triangular prisms. The milk could pour from a hole in one corner of the top. What is the volume of the container of milk shown at the right? Write a complete sentence to explain how you used the formula for the volume of a prism to find the volume of milk in the container.

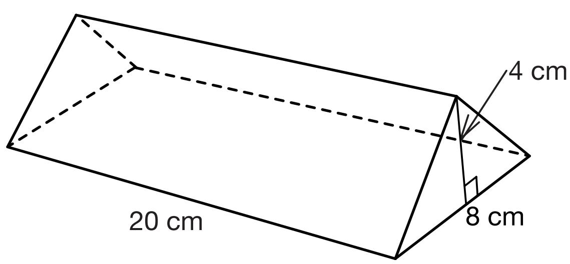


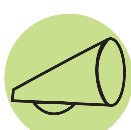
9. Find the surface area of the milk container. Write a complete sentence that explains how you found the answer.

10. What is the volume of a box of your favorite cereal? Measure the length, width, and height of the box. Then calculate the volume. Write your answer using a complete sentence.

11. Find the surface area of a box of your favorite cereal. Write a complete sentence that explains how you found the answer.

12. A certain kind of imported chocolate comes in packages that are shaped like triangular prisms, as shown below. What is the volume of chocolate that the container could hold? Write your answer using a complete sentence.





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12.3 The Playground Olympics

Volumes and Surface Areas of Cylinders

Objectives

In this lesson, you will:

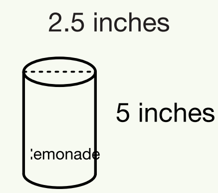
- Find volumes of cylinders.
- Find surface areas of cylinders.

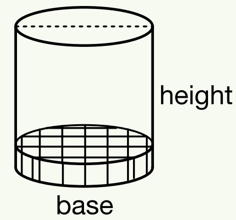
Key Terms

- volume
- surface area

Problem 1 Lemonade Dilemma

Walnut Grove Middle School holds an annual event called the Playground Olympics. This year the student council is planning to sell lemonade during the event. The group wants to use a drink dispenser instead of selling cans of lemonade. They decide to measure the height and diameter of a typical lemonade can. The group wants to find the volume of the can.





A. To find the volume of a cylinder, imagine filling the cylinder completely with unit cubes and parts of unit cubes. The volume of a cylinder is the number of unit cubes that it holds. We can use the same method that we used for finding the volume of a prism. That is, we can multiply the area of the base (one layer of unit cubes) by the height (the number of layers of unit cubes that the cylinder holds).

Because the base of a cylinder is always a circle, the area of the base is πr^2 where r is the radius. The formula becomes

Volume of a cylinder = $B \times h = \pi r^2 h$.

Find the volume of lemonade that the can holds. Use 3.14 for π.

B. The student council decides to design a cylindrical cup that will hold one half as much lemonade as a can and also a cylindrical cup that will hold twice as much lemonade. Form a group with another partner team. In your group, design cylindrical cups with different dimensions that will have one half of the volume of a typical can and cylindrical cups with different dimensions that will hold twice the volume of a typical can.

C. Share your results with other groups in your class. Record the different cup sizes that were found.

Investigate Problem 1

1. The class president thinks that the student council should also design a cup that holds 3 times as much as a typical lemonade can. What possible dimensions could this cup have? Draw and label your design for this "Giant Gulp" cup.

2. At the Playground Olympics, the Pep Club plans on selling souvenir tennis balls with the school logo stamped on each ball. Tennis balls are 6 centimeters in diameter. Design several containers to hold the tennis balls. Sketch each container and label the dimensions.

   Design a cylindrical container that will hold 1 tennis ball.

   Design a cylindrical container that will hold 4 tennis balls.

   Design a cylindrical container that will hold 8 tennis balls.

   Design a container in the shape of a rectangular prism that will hold 1 tennis ball.

   Design a container in the shape of a rectangular prism that will hold 4 tennis balls.

   Design a container in the shape of a rectangular prism that will hold 8 tennis balls.

   Complete the table below.

Number of Tennis Balls	Cylinder			Rectangular Prism		
	Height (cm)	Radius (cm)	Volume (cm^3)	Height (cm)	Area of Base (cm^2)	Volume (cm^3)
1						
4						
8						

3. Form a group with another partner team. Compare your containers with your group's containers. Did everyone in your group design the same containers? How were the containers different?

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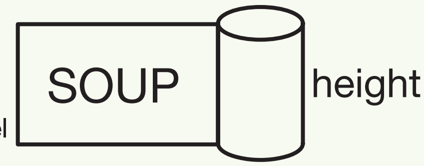
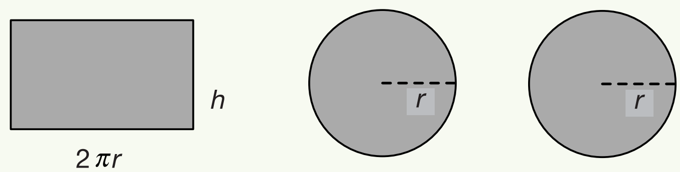
Problem 2 Can Do!

Recall from Lesson 12.2 that the surface area is the sum of the areas that form the surface of a three-dimensional figure. The label of a can covers the surface of the can that is not circular. If you cut the label from a can, you can see that the label is a rectangle.



A. The width of the rectangle is the height of the can. Because the label wraps around the can, the length of the rectangle is the circumference of the can. The surface area of the can is the area of the rectangle plus the area of the two circular bases.



Area = $2\pi rh$ Area = πr^2 Area = πr^2

Surface Area of a Cylinder = $2(\pi r^2) + 2\pi rh$

Find the surface area of a can with a radius of 3.5 inches and a height of 6 inches. Use 3.14 for π.

B. Manufacturers try to design containers that hold a maximum amount of product (soft drinks, soup, etc.) but use a minimum amount of surface material (plastic, paper, tin, etc.) They measure a can's efficiency rating by dividing the surface area (SA) by the volume (V) and determining the ratio $\frac{SA}{V}$.

In your group, bring several cans from home or ask the workers in the cafeteria to borrow some cans for this investigation. Measure the radius and height of each can. Then determine the volume and surface area of each can. Finally, calculate the ratio $\frac{SA}{V}$. Use the table below to organize your work.

Type of Can	Height (cm)	Radius (cm)	Volume (cm^3)	Surface Area (cm^2)	$\frac{SA}{V}$

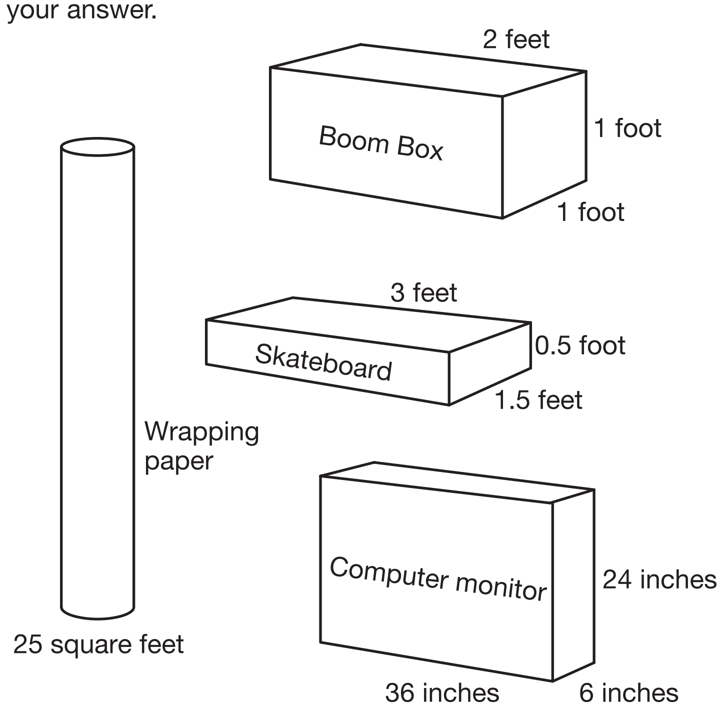
Investigate Problem 2

1. Which cans in part (B) were the most efficient? Which cans were the least efficient? Write complete sentences to explain your findings.

2. Label each can with its efficiency rating. Line the cans up across the front of the classroom from least efficient to most efficient. What has the greatest effect on the volume of a cylinder, doubling the radius or doubling the height? Use a complete sentence to explain your reasoning.

   What has the greatest effect on the surface area of a cylinder, doubling the radius or doubling the height? Use a complete sentence to explain your reasoning.

3. Wrapping paper comes on rolls and is measured in square feet. If you have the three presents below to wrap for the Playground Olympics prizes, will you have enough paper? Use complete sentence to write a paragraph that explains how you found your answer.



12.4

The Rainforest Pyramid

Volumes of Pyramids and Cones

Objectives

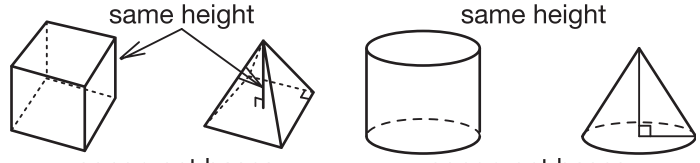
In this lesson, you will:

- Find volumes of pyramids.
- Find volumes of cones.

Key Terms

- pyramid
- cone

In Lesson 12.1, we looked at pyramids and cones. If a prism and a pyramid have congruent bases and the same height, will they have the same volume? If a cylinder and a cone have congruent circular bases and the same height, will they have the same volume?



Problem 1 Selling Popcorn

A middle school class is selling popcorn to raise money for a class trip to the Rainforest Pyramid in Galveston, Texas. The students would like to use one of the four types of containers shown above to hold the popcorn. Some of the students think that if the heights of the four containers are the same, then the volumes of the four containers will be the same. Other students think that there may be a difference.

In your group, investigate the volumes of these pairs of solids. You will need a hollow prism and a hollow pyramid with the same base and height, and a hollow cylinder and a hollow cone with the same base and height. You can use rice, sand, or water to investigate these volumes. Be sure to use a box or another container under your shapes to catch the spills. Follow the steps below.

Step 1: Fill your pyramid with sand, rice, or water. Then very carefully pour the contents of the pyramid into the prism. About how much of your prism is filled?

Repeat Step 1 until the prism is filled. How many times did you pour the volume of your pyramid into the prism to fill the prism?

Step 2: Fill your cone with sand, rice, or water. Very carefully pour the contents into the cylinder. About how much of the cylinder is filled?

Repeat Step 2 until the cylinder is filled. How many times did you pour the volume of your cone into the cylinder to fill the cylinder?

Investigate Problem 1

1. Compare your group's results with the results of other groups in your class. Were the results similar for the two pairs of solids? Write a complete sentence to explain.

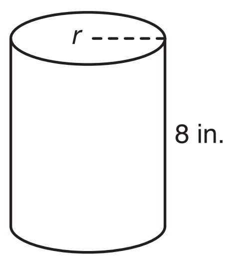
2. Complete the statements below to summarize your investigation in Problem 1.

   The volume of a pyramid is _____ of the volume of a prism if the pyramid and the prism have congruent bases and the same height.

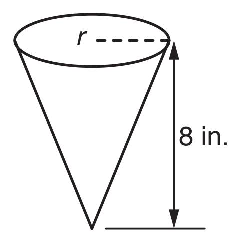
   The volume of a cone is _____ of the volume of a cylinder if the cone and the cylinder have congruent circular bases and the same height.

3. An organization's profit is the difference between its income and its expenses. Which price for the popcorn will bring the most profit? Use complete sentence to explain your reasoning.

Jumbo size popcorn $3



Medium size popcorn $2



4. Determine a fair pricing scheme for selling popcorn in a cylindrical container and a cone-shaped container if the containers have the same height and congruent bases. Write your answer using complete sentences.

5. What would be a fair pricing scheme for selling popcorn in a rectangular prism-shaped container and a pyramid-shaped container that have the same height and congruent bases? Write your answer using complete sentences.

Problem 2 Visiting the Rainforest Pyramid

In Problem 1, we discovered that if a rectangular prism and a pyramid have congruent bases and the same height, then the pyramid's volume is $\frac{1}{3}$ of the rectangular prism's volume. This is true for all pyramids. So,

$$\text{Volume of a pyramid} = \frac{1}{3}B \times h$$

where B is the area of the base and h is the height.

The Rainforest Pyramid in Galveston, Texas, is a building that is 100 feet high and has a square base with sides that are 200 feet in length. What is the volume of this building? Show your work below. Then write your answer using a complete sentence.

Investigate Problem 2

1. The height of the Great Pyramid of Giza is 480 feet and its square base has a side length of 750 feet. What is the volume of the Great Pyramid? Show your work at the left. Then write your answer using a complete sentence.

2. The Transamerica Building in San Francisco, California, is also a square pyramid. It is 853 feet tall and its square base has an area of 2025 square feet. Find the volume of the Transamerica Building to the nearest cubic foot. Show your work at the left. Then write your answer using a complete sentence.

3. The pyramid arena in Memphis, Tennessee, is 321 feet tall and has a square base that is 300 feet on each side. What is the volume of this arena? Show your work at the left. Then write your answer using a complete sentence.

Take Note

The **height** of a cone is the length of the segment from the common vertex to the base such that the segment is perpendicular to the base.

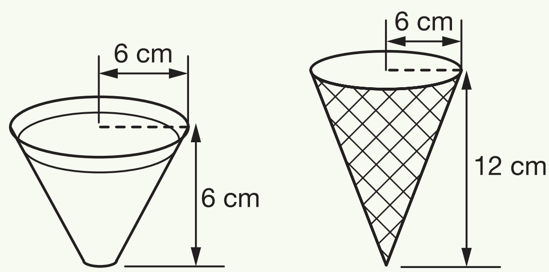
Problem 3 Cups or Cones?

In Problem 1, we discovered that if a cone and a cylinder have congruent bases and the same height, then the cone's volume is $\frac{1}{3}$ of the cylinder's volume. This is true for all cones. So,

$$\text{Volume of a cone} = \frac{1}{3}B \times h = \frac{1}{3}\pi r^2 h$$

where r is the radius and h is the height.

A. The middle school class wants to plan a trip for next year, so the class president decides that it would be much more profitable to sell soft ice cream on hot days rather than to continue selling popcorn. The class needs to order containers to hold the ice cream, so they order the cups and cones, shown below.



The class president says, "Let's charge \$1 for the cup and \$2 for the cone, because the cone is twice as tall as the cup." The class treasurer says, "No way—both the cone and the cup have the same volume, so we have to charge the same amount, \$2 for each." Who is right, the president or the treasurer? Find the volume of the cup and the volume of the cone.

Volume of cup =

Volume of cone =

B. What would be a fair price to charge for each? Write a complete sentence to explain your answer.

C. Change the dimensions of the cup so that the new cup has the same volume as the cone.

New dimensions of cup:

height = diameter =

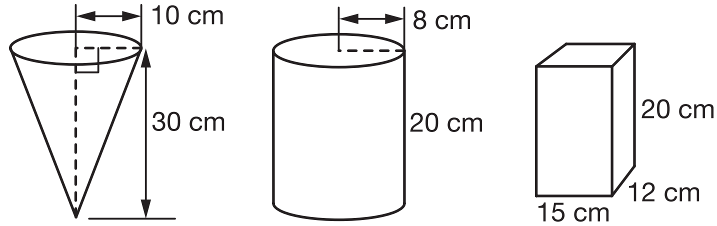
D. Change the dimensions of the cone so that the new cone has the same volume as the original cup.

New dimensions of cone:

height = diameter =

Investigate Problem 3

1. The fourth grade class at the elementary school decided that the middle school was selling ice cream so they should sell popcorn. They gathered all of the old containers that no one else was using. They found cones, cylinders, and boxes shaped like rectangular prisms.



The prices they decided on were:

cone: \$2.50

cylinder: \$3.75

rectangular prism: \$3.50

Which container gives customers the best buy for their money? Use the volume of each container to explain your answer. Use complete sentences in your explanation.

Volume of cone:

Volume of cylinder:

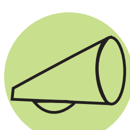
Volume of rectangular prism:

12

Investigate Problem 3

2. Use an 8.5-inch by 11-inch sheet of paper to design a container. The container should have the greatest volume possible. You may cut and tape the paper together to form a three-dimensional figure. You may use only one sheet of paper.

   Record all of the dimensions of your container and the volume of your container. Be sure to write down any diagrams, tables, or equations that you used to help you solve the problem.



Each group should present their container to the class, explaining their process and their conclusions.

Which group's container had the largest volume?

What was the shape of the container with the largest volume?

What was the volume of this container?

What patterns did you notice with all of the containers designed? Write your answer using complete sentences.

12.5 What on Earth?

Volumes and Surface Areas of Spheres

Objectives

In this lesson, you will:

- Find volumes of spheres.
- Find surface areas of spheres.

Key Terms

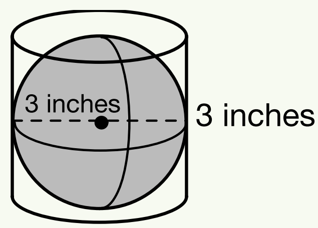
- sphere
- hemisphere

A **sphere** is a very common geometric figure. You may know it by its most common name of "ball." You can describe a sphere by giving its radius.

Problem 1 Earth's Volume

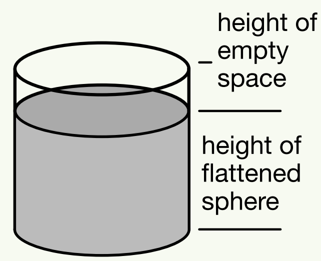
Earth is not a perfect **sphere**, but it has been formed by gravitational forces into a spherical shape. Scientists in ancient Greece and Egypt determined that Earth was round by observing the shadow of Earth as it passed across the moon during a lunar eclipse. Eratosthenes, a scientist from Alexandria, Egypt, who lived during the third century B.C., actually estimated the circumference of Earth.

A. To determine the volume of a sphere, let's compare it to the volume of a cylinder. Using clay or modeling dough, make a sphere with a diameter of 3 inches.



Wrap a strip of plastic around the sphere (a transparency sheet will work). Then trim the plastic so that it is as tall as the sphere you made.

Tape the plastic together tightly to form a cylinder with an open top and bottom.



Next, flatten the sphere so that it fits snugly in the bottom of the cylinder.

Measure the following dimensions.

height of the cylinder = ______________

height of the flattened sphere = ______________

height of the empty space in the cylinder = ______________

What relationship do you notice?

B. The volume of the sphere is about what fractional part of the volume of the cylinder? Write your answer using a complete sentence.

Investigate Problem 1

1. If your experiment worked out perfectly, you would find that the volume of a sphere is $\frac{2}{3}$ of the volume of a cylinder with the same diameter.

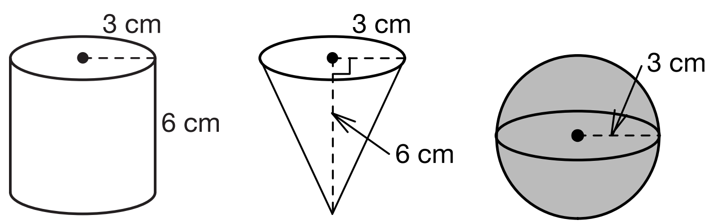
   Volume of a sphere = $\frac{2}{3}$(Volume of a cylinder) = $\frac{2}{3}(\pi r^2 h)$

   Remember that the height of the cylinder is equal to the diameter of the sphere, so the height of the cylinder is $2r$. We can substitute $2r$ for the height into the equation.

   Volume of a sphere = $\frac{2}{3}\pi r^2(2r) = \frac{4}{3}\pi r^3$

   Find the volume of the sphere that you made in Problem 1.

2. Think about the earlier investigation that you did in which you investigated the relationship between a cone and a cylinder. Both cones and spheres are related to cylinders. Find the volumes of the three figures below. Use 3.14 for π.



   Volume of cylinder: ______________ cubic centimeters

   Volume of cone: ______________ cubic centimeters

   Volume of sphere: ______________ cubic centimeters

3. How do the volumes compare? Use mathematics to explain. Write your answer using a complete sentence.

4. Earth has a diameter of about 7926 miles and a surface area of about 197 million square miles.

   What is the circumference of Earth at the equator?

   Circumference of Earth: __________________________ miles

   What is the volume of Earth?

   Volume of Earth: __________________________ cubic miles

Investigate Problem 1

5. Eratosthenes estimated the circumference of Earth to be about 28,750 miles. How far off was his estimate? Write your answer using a complete sentence.

6. **Math Path: Hemisphere**

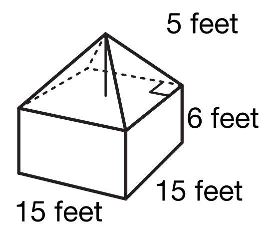
   A **hemisphere** is exactly half of a sphere. An igloo is shaped like a hemisphere. What is the volume of the space of an igloo with an inside diameter of 50 feet?

   If a tepee shaped like a cone has the same volume as the igloo, what are the possible dimensions of the tepee?

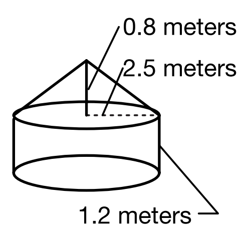
   A nomadic tent used in the Sahara desert is shaped like a triangular prism. If this tent had the same volume as the igloo, what would its dimensions be?

   An adobe house is shaped like a rectangular prism. What would the dimensions of the adobe house be if the house had the same volume of space as the igloo?

7. A Native American hogan was a more permanent dwelling than a tepee, which could be easily rolled up and transported. A hogan is shaped like a rectangular prism with a rectangular pyramid on top. What is the volume of a typical hogan as shown at the right?



8. A yurt is a dwelling used by nomadic tribes in Mongolia. It is very weather-resistant and can be easily transported by folding it. It is shaped like a cylinder with a cone on top. What is the volume of a typical yurt as shown at the right? Use 3.14 for π.



Problem 2 Another House on Earth

A geodesic dome is used by many people as an energy-efficient home. Richard Buckminster Fuller created the geodesic dome. A geodesic dome is approximately a hemisphere.

A. What is the volume of a geodesic dome that has a circular base with a diameter of 40 feet? Write your answer using a complete sentence and explain your reasoning.

B. We want to determine the amount of material needed to make a geodesic dome roof. We need to find the surface area of the geodesic dome, a hemisphere. We cannot flatten out the surface of a sphere to easily measure its surface area, but we can investigate the formula for the surface area of a sphere.

Choose a ball similar in size to a tennis ball. Measure its diameter.

Construct 5 paper circles with the same diameter as the ball. Tape the circles to the surface of the ball with no overlaps. This is somewhat difficult and you will need to cut the circles to fill in the gaps. Use each circle's area completely before you start using another circle.

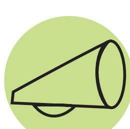
How many circles did you use to completely cover the ball with no overlap?

Investigate Problem 2

1. If this wrapping of the sphere is done perfectly, you will find that it takes exactly 4 circles to cover the sphere. The formula for the surface area of a sphere is 4 times the area of a circle with the same radius.

Surface area of a sphere = $4\pi r^2$

Find the surface area of the geodesic dome in part (A) above.



2. Find the volume and surface area of each type of sports ball.

Type of Ball	Diameter	Volume	Surface Area
ping-pong	40 millimeters		
golf ball	1.5 inches		
baseball	7.5 centimeters		
bowling ball	8.6 inches		

Engineers and Architects

Nets and Views

Objectives

In this lesson, you will:

- Design nets for three-dimensional figures.
- Construct side, front, and top views of three-dimensional objects.

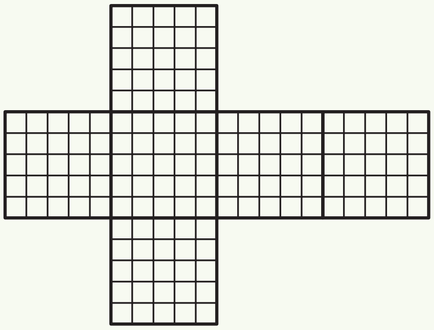
Key Terms

- net

Packaging engineers design crates, barrels, and other containers by designing nets. A **net** is a two-dimensional pattern that you can fold to form a three-dimensional figure.

Problem 1 Creating Shipping Boxes

A shipping company is shipping 1000 computer monitors by boat. Each monitor is in the shape of a cube with a side length of 1 foot. A well-designed box will hold a computer monitor in place with no extra space. One **net** for the box is shown below.



A. Help the company design a box to hold the computer monitor by drawing several other nets that would fold to make a box. Use graph paper to design your nets.

B. Cut out each net that you designed in one piece. Fold each net into a 1 cubic foot box. Form a group with another partner team. How many different nets were produced by your group?

C. Nets can also help us to calculate the surface area of three-dimensional figures. A net is a two-dimensional representation of the surface area of a figure.

The shipping company must also design steel shipping containers (with lids) to hold the shipping boxes. They need to have three sizes of containers. A small container will hold 8 boxes, a medium container will hold 24 boxes, and a large container will hold 36 boxes. The containers need to be designed to use the steel as efficiently as possible. Design the most efficient container for each of the three sizes. Write a complete sentence that explains why your design is the most efficient.

Problem 1 Creating Shipping Boxes

D. Sketch the nets that you designed.

E. For each container, record its dimensions, the surface area of each face of the container, and the total surface area of each container in the table.

Number of Boxes	Dimensions of Most Efficient Container	Total Surface Area (square feet)
8		____ + ____ + ____ + ____ + ____ + ____ + = _______
24		____ + ____ + ____ + ____ + ____ + ____ + = _______
36		____ + ____ + ____ + ____ + ____ + ____ + = _______

Investigate Problem 1

1. Bring in an empty cereal box from home. Cut it apart very carefully and fold it flat. This is the net that the cereal manufacturer designed for your brand of cereal. Measure the dimensions of the net.

What is the surface area of your cereal box?
_______ square _______________

What is the volume of your cereal box?
_______ cubic _______________

2. Bring in a can of food from home. Measure the dimensions of your can. Then make a net for your can.

What is the surface area of your can?
_______ square _______________

What is the volume of your can?
_______ cubic _______________

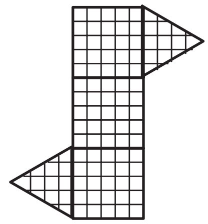
3. Design a net for a cylinder that would have enough volume to hold the contents of your cereal box. Label the cylinder's dimensions.

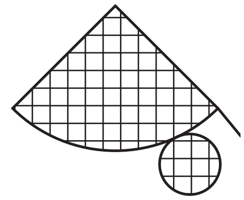
What is the surface area of your cylinder?
_______ square _______________

What is the volume of your cylinder?
_______ cubic _______________

Investigate Problem 1

4. For each net below, name the solid that can be formed.



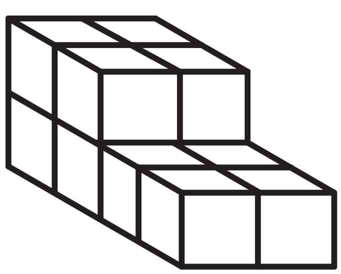


5. Draw four nets for four different geometric figures. Label the dimensions of the net. Trade nets with your partner and name the figure represented by the nets. Check each other's work.

Problem 2 Front, Top, and Side

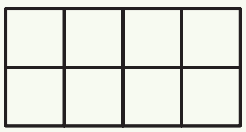
Although we live in a three-dimensional world, we often represent objects in our world using two-dimensional images, drawings, or pictures. Architects create plans for buildings on blueprints that show every aspect of the building—front, back, right side, left side, top, and bottom. Construction crews use these two-dimensional plans to construct the three-dimensional structure.

Use sugar cubes or some other type of cube to construct a model of the building shown at the right. How many cubes were used to construct this model?

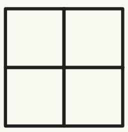


Imagine that you are standing directly in front of this building. What do you see? Choose the view that represents the front view and label it.

Is the back view the same?

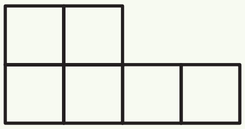


Imagine that you are standing above the building looking down. What is the top view of the building? Choose the view that represents the top view and label it.



Is this the same as the bottom view? The bottom view is often called the floor plan.

Now imagine that you are looking directly at the left side. What is this left side view? Choose the view that represents the side view and label it.

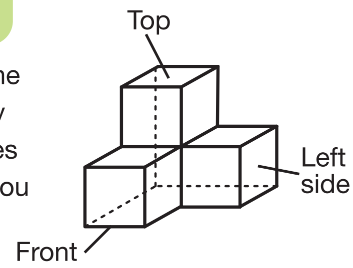


Is the right side view the same?

What is the surface area of this building in square units?

Investigate Problem 2

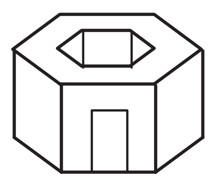
1. Draw the six views of the model of the building shown at the right. You may want to model the building with cubes to help you visualize the views that you cannot directly see.



Top **Front** **Left side**

Right side **Bottom** **Back**

2. An architect designed this building called "hexagon." Draw the six views of the building.



Top **Front** **Left side**

Right side **Bottom** **Back**

12.7 Double Take

Similar Solids

Objectives

In this lesson, you will:

- Compare volumes of similar solids.
- Compare surface areas of similar solids.

Key Terms

- scale factor
- similar solids

In Lesson 9.4 we learned that corresponding angles in similar figures are congruent and that corresponding sides are proportional. Recall that figures can be scaled up or scaled down by a *scale factor.*

Problem 1 Peanuts and Sports Drinks

Three-dimensional objects can also be **scaled** up or scaled down. A model car is a scaled-down version of the real object. The model is mathematically similar to the original object. Every detail is in proportion.

A. **Similar solids** are solids that have the same shape but not necessarily the same size. Use what you know about cubes, spheres, and cylinders to decide whether each statement is true or false. For each statement, write a complete sentence that explains your reasoning.

Not all cubes are similar.

All prisms are similar.

All spheres are similar.

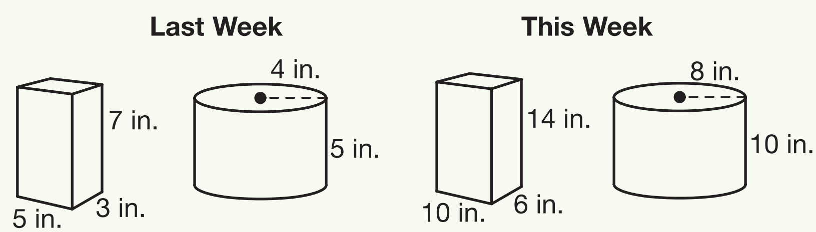
Not all cylinders are similar.

Problem 1 Peanuts and Sports Drinks

B. Two polyhedrons are similar if their corresponding bases are similar and the lengths of their corresponding heights are proportional. Two cones or cylinders are similar if the ratio of their radii equals the ratio of their heights.

Bill and Rick own a concession stand near a baseball park. Bill and Rick sell a sports drink and peanuts at the baseball games. Every week they try different marketing strategies to attract more business.

This week they decide to give everyone twice as much sports drink and twice as many peanuts for the same price as last week. They order boxes and cups with dimensions that are twice the dimensions of the boxes and cups that they used the week before. By super-sizing everything, they hope to sell twice as much. Here are the dimensions of last week's and this week's containers.



Bill took one look at the new containers and said, "They are huge. Are you sure that they hold twice as much?" Rick said, "I'm positive. I scaled them up by 2. Every dimension is twice as much." All the spectators, even the players, wanted to buy these new super-sized containers for the old prices.

Before second inning was over, Bill and Rick sold out and they had a lot less money than they had at any other game. They were puzzled and couldn't figure out what went wrong. The baseball coach was also their math teacher. He asked them to calculate the volume of last week's containers and the volume of the new scaled-up containers from this week. Help Bill and Rick find the volume of each container from last week and this week.

Volume of last week's peanut container: ________________

Volume of scaled-up peanut container: ________________

Volume of last week's sports drink container: ________________

Volume of scaled-up sports drink container: ________________

12

Investigate Problem 1

1. How do the volumes compare? Use a complete sentence in your answer.

2. Was the volume also scaled up by 2? Use a complete sentence to explain.

3. What should Bill and Rick have charged for their scaled-up containers? Write a complete sentence to explain your reasoning.

4. For both the peanut and the sports drink containers, write the ratio of the volume of last week's containers to the volume of the super-sized containers.

Peanuts: $\frac{\text{Volume of last week's container}}{\text{Volume of scaled-up container}} = \frac{\square}{\square} = \frac{\square}{\square}$

Sports drink: $\frac{\text{Volume of last week's container}}{\text{Volume of scaled-up container}} = \frac{\square}{\square} = \frac{\square}{\square}$

5. How many of last week's containers fit into the scaled-up containers for peanuts?

How many of last week's containers fit into the scaled-up containers for sports drink?

How does this number relate to the scale factor 2? Write a complete sentence that explains your reasoning mathematically.

6. Are last week's containers and the scaled-up containers similar? Are the ratios of the corresponding dimensions the same?

Ratios of peanuts containers:

length: $\frac{5 \text{ in.}}{10 \text{ in.}} = \frac{\square}{\square}$

width: $\frac{3 \text{ in.}}{6 \text{ in.}} = \frac{\square}{\square}$

height: $\frac{7 \text{ in.}}{14 \text{ in.}} = \frac{\square}{\square}$

Ratio of sports drink containers:

height: $\frac{5 \text{ in.}}{10 \text{ in.}} = \frac{\square}{\square}$

radius: $\frac{4 \text{ in.}}{8 \text{ in.}} = \frac{\square}{\square}$

Investigate Problem 1

7. Bill and Rick had to pay four times as much for the scaled-up containers. That also cut into their profits. Bill thought that maybe they had been overcharged, but Rick thought that they should calculate the surface area of the four containers before they complained.

   Surface area of last week's peanut container: _______________

   Surface area of scaled-up peanut container: _______________

   Surface area of last week's sports drink container: ____________

   Surface area of scaled-up sports drink container: _____________

8. How do the surface areas compare? Use a complete sentence in your answer.

9. Was the surface area of the scaled-up containers twice the surface area of last week's containers? Use a complete sentence to explain.

10. For both the peanut and the sports drink containers, write the ratio of the surface area of last week's containers to the surface area of the super-sized containers.

Peanuts: $\dfrac{\text{Surface area of last week's container}}{\text{Surface area of scaled-up container}} = \dfrac{\square}{\square} = \dfrac{\square}{\square}$

Sports drink: $\dfrac{\text{Surface area of last week's container}}{\text{Surface area of scaled-up container}} = \dfrac{\square}{\square} = \dfrac{\square}{\square}$

11. How do the numbers in Question 10 relate to the scale factor 2? Write a complete sentence that explains your reasoning mathematically.

12. If Bill and Rick had truly wanted to enlarge the containers so that they would hold twice as much, what should they have done? Design a peanut container and a sports drink container that do hold twice as much. Record the dimensions and sketch the containers at the left.

13. Form a group with another partner team. In your group, discuss how the volume and the surface area of a geometric solid change as each of its dimensions is doubled, tripled, quadrupled, and so on.

Looking Back at Chapter 12

Key Terms

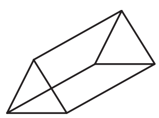
prism • p. 381
face • p. 381
base • p. 381
height • p. 381
vertex • p. 381
edge • p. 381
solid • p. 382
polyhedron • p. 382
pyramid • p. 382
cylinder • p. 382
cone • p. 383
sphere • p. 383
cube • p. 385
volume • p. 386
surface area • p. 387
sphere • p. 401
hemisphere • p. 403
net • p. 405
scale factor • p. 409
similar solids • p. 409

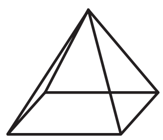
Summary

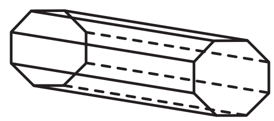
Identifying Three-Dimensional Figures (p. 381)

Prisms and pyramids are three-dimensional solids called polyhedrons and are named for the shape of their bases. Other solids are cylinders, cones, and spheres.

Examples


Triangular Prism

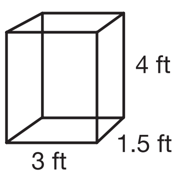

Rectangular Pyramid


Octagonal Prism

Finding Volumes of Prisms (p. 386)

To find the volume of a prism, multiply the area of the base by the height of the prism.

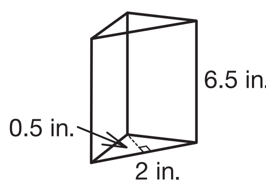
Examples



Volume = $\ell \times w \times h$

Volume = $3 \times 1.5 \times 4$

= 18 cubic feet



Volume = $B \times h$

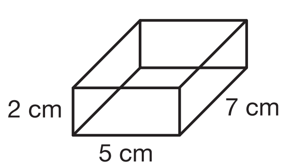
Volume = $\left(\frac{1}{2} \times 2 \times 0.5\right) \times 6.5$

= 3.25 cubic inches

Finding Surface Areas of Rectangular Prisms (p. 387)

To find the surface area of a rectangular prism, find the sum of the areas of the surfaces of the prism.

Example

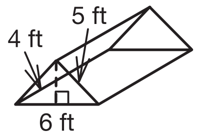


Area of bases = $2(2 \times 5) = 2 \times 10 = 20$ square centimeters
Area of sides = $2(2 \times 7) = 2 \times 14 = 28$ square centimeters
Area of sides = $2(5 \times 7) = 2 \times 35 = 70$ square centimeters
Surface Area = 20 + 28 + 70 = 118 square centimeters

Finding Surface Areas of Triangular Prisms (p. 388)

To find the surface area of a triangular prism, find the sum of the areas of the surfaces of the prism.

Example



Area of bases = $2(\frac{1}{2} \times 6 \times 4) = 2 \times 12 = 24$ square feet

Area of sides = $2(5 \times 8) = 2 \times 40 = 80$ square feet

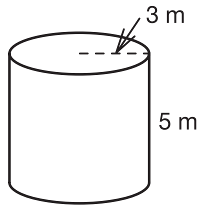
Area of bottom = $6 \times 8 = 48$ square feet

Surface Area = $24 + 80 + 48 = 152$ square feet

Finding Volumes of Cylinders (p. 391)

To find the volume of a cylinder, multiply the area of the circular base by the height of the cylinder. You can use 3.14 for π.

Example



Volume = $\pi r^2 h$

Volume = $\pi (3)^2(5)$

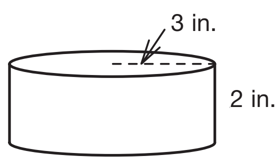
$\approx 3.14(9)(5)$

$= 141.3$ cubic meters

Finding Surface Areas of Cylinders (p. 393)

To find the surface area of a cylinder, find the sum of the areas of the circular bases and the area of the rectangle that forms the side of the cylinder. You can use 3.14 for π.

Example



Area of bases = $2\pi r^2 \approx 2(3.14)(3^2) = 56.52$ sq. in.

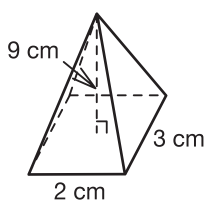
Area of rectangle = $2\pi rh \approx 2(3.14)(3)(2) = 37.68$ sq. in.

Surface Area = $37.68 + 56.52 = 94.2$ sq. in.

Finding Volumes of Pyramids (p. 396)

To find the volume of a pyramid, multiply one third of the area of the base by the height.

Example



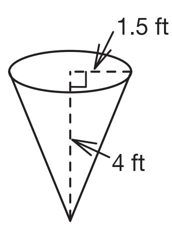
Volume = $\frac{1}{3}B \times h = \frac{1}{3}(2 \times 3) \times 9$

$= 18$ cubic centimeters

Finding Volumes of Cones (p. 396)

To find the volume of a cone, multiply one third of the area of the circular base by the height of the cone. You can use 3.14 for π.

Example



Volume = $\frac{1}{3}B \times h = \frac{1}{3}\pi r^2 h$

Volume $\approx \frac{1}{3}(3.14)(1.5)^2(4)$

$= 9.42$ cubic feet

Finding Volumes and Surface Areas of Spheres (p. 401)

To find the volume of a sphere, use the formula:

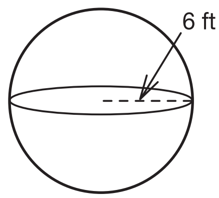
Volume of a sphere = $\frac{4}{3}\pi r^3$.

To find the surface area of a sphere, use the formula:

Surface Area = $4\pi r^2$.

You can use 3.14 for π .

Example



Volume = $\frac{4}{3}\pi r^3$

Volume ≈ $\frac{4}{3}(3.14)(6)^3$

= 904.32 cubic feet

Surface area = $4\pi r^2$

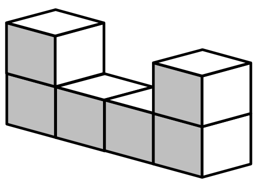
Surface area ≈ $4(3.14)(6)^2$

= 452.16 square feet

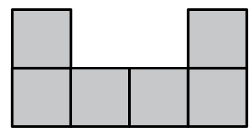
Constructing Views of Three-Dimensional Objects (p. 407)

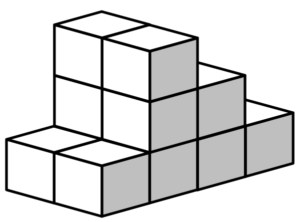
To construct different views of a three-dimensional object, imagine that you are looking directly at the top, front, or side of the object.

Examples

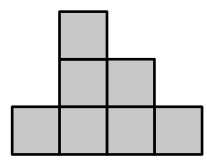


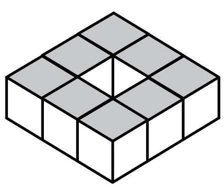
Side View



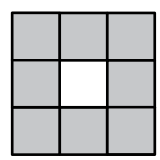


Front View





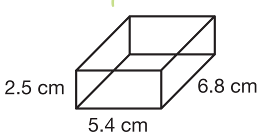
Top View

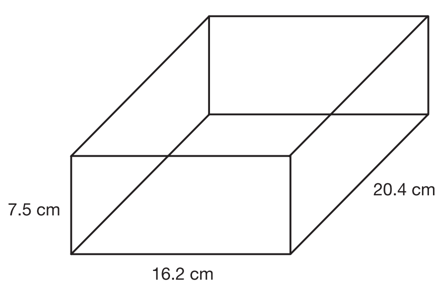


Determining Whether Solids are Similar Solids (p. 409)

To determine whether two solids are similar, find the ratios of their corresponding dimensions. If the ratios of all corresponding dimensions are the same, the solids are similar.

Example





$\frac{2.5 \text{ cm}}{7.5 \text{ cm}} = \frac{1}{3}$

$\frac{5.4 \text{ cm}}{16.2 \text{ cm}} = \frac{1}{3}$

$\frac{6.8 \text{ cm}}{20.4 \text{ cm}} = \frac{1}{3}$

Because the ratios are the same, the solids are similar.