Looking Ahead to Chapter 10

Focus In Chapter 10, you will find perimeters of rectangles, find circumferences of circles, and find the areas of rectangles, circles, parallelograms, triangles, trapezoids, and composite figures. You will find squares and square roots of numbers, and use the Pythagorean theorem to solve problems.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 10.

Use mental math to find the sum.

1. 27 + 94
2. 13 + 45
3. 9 + 5 + 21

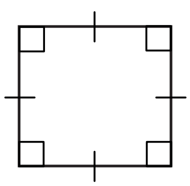
Find the product.

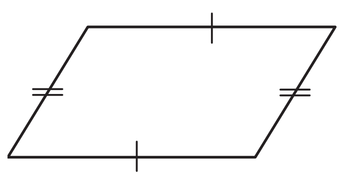
4. 3.4×5.7
5. 9.1×2.6
6. 1.52×7.8

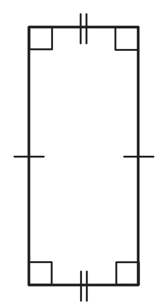
Evaluate the power.

7. 9^2
8. 20^2
9. 45^2

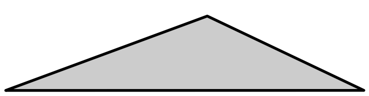
Write as many names as you can for the quadrilateral.

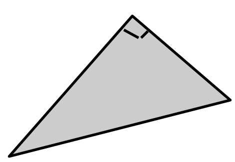
10. 

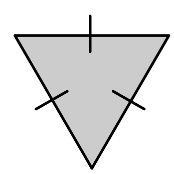
11. 

12. 

Your aunt is using the triangle to make a quilt. Describe the triangle as acute, right, or obtuse.

13. 

14. 

15. 

Key Terms

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Area and the Pythagorean Theorem

A skate park may contain half pipes, quarter pipes, banked ramps, stairs, and other objects so that skateboarders can do tricks. In Lesson 10.3, you will use what you know about area to help design a skate park for a city.

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10.1 All Skate!

Perimeter and Area

Objectives

In this lesson, you will:

- Find perimeters of rectangles.
- Find areas of rectangles.
- Determine the effect on perimeter and area of changing dimensions.

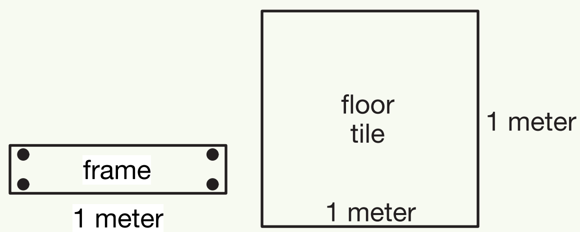
Key Terms

- perimeter
- area

The **perimeter** of a closed figure is the distance around the figure. Perimeter is measured in linear units, such as feet or centimeters. The **area** of a closed figure is the number of square units needed to cover the figure. Area is measured in square units, such as square feet or square centimeters.

Problem 1 Skating Rink

At Starlight Middle School, students construct a roller skating rink in the shape of a rectangle each day during lunch period. They form the frame around the outside of the rink with interlocking framing pieces that are each one meter long and form the floor with tiles that each cover one square meter.



Because the students don't want to always skate in the same configuration, they build a different rectangular rink each day. The school has 36 flooring tiles. Complete the table to find the number of different rectangular rinks that the students can construct using the 36 tiles. For each rectangular rink, how many pieces of framing will they need around the outside of the rink to hold it together? If you need to, draw diagrams of the rinks in the space at the left.

Rectangles with an Area of 36 Square Meters		
Length	**Width**	**Number of Framing Pieces Needed (Perimeter)**

Investigate Problem 1

1. What patterns do you notice in the table in Problem 1? Compare your thinking with your partner's.

2. Work with your partner to write a formula that you can use to find the area of any rectangle.

3. Work with your partner to write a formula that you can use to find the perimeter of any rectangle.

4. Form a group with another partner team. Compare the formulas that you found for area and perimeter.

5. On Monday, for the individual speed-skating competition, it is important to have as much space as possible for students to stand around the edges of the rink to watch the speed trials. Which rectangle should the students build so that the standing room around the edges of the rink is as large as possible (that is, which rectangle has the maximum perimeter)?

6. On Wednesday, there is an "everybody skate" and very few students watch. Which rectangle should the students build so that the standing room around the edges of the rink is as small as possible (that is, which rectangle has the minimum perimeter)? Write a complete sentence to explain your thinking.

7. On Friday, the nearby elementary school borrows some of the framing pieces so that the students are left with only 24 pieces of framing. Find the number of different sizes of rectangular skating rinks the students can build using all 24 pieces of framing. How many flooring tiles will they need to use? Complete the table.

Rectangles with a Perimeter of 24 Meters		
Length	**Width**	**Number of Flooring Tiles Needed (Area)**

Investigate Problem 1

8. What patterns do you notice in the table in Question 7? Write your answer using a complete sentence.

9. Do the formulas that you found in Questions 2 and 3 for area and perimeter work for these rectangles? List two examples for area and two examples for perimeter showing whether they work or not.

10. Which rectangle should the students build if they want the greatest amount of skating area? Write your answer using a complete sentence.

11. What are the length and width of the rink that the students should build if they want the least amount of skating area for individual practice? Write your answer using a complete sentence.

12. Use the results of Questions 5, 6, 10, and 11 to answer each question. Use complete sentences.

Given a rectangle with a fixed area, how can you maximize the perimeter and how can you minimize the perimeter?

Given a rectangle with a fixed perimeter, how can you maximize the area and how can you minimize the area?

13. Suppose that the students have 48 square tiles and use all of them to build a rectangular rink. What are the length and width of the rink with the largest perimeter? What are the length and width of the rink with the smallest perimeter?

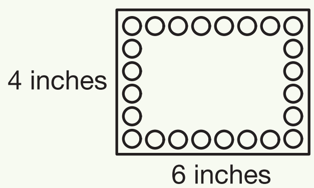
14. Suppose that the students have 48 framing pieces and use all of them to build a rectangular rink. What are the length and width of the rink with the largest area? What are the length and width of the rink with the smallest area?

In Lesson 9.4, you learned about scale factors and similar figures. Let's look at what happens to the perimeter and area of similar figures.

Problem 2 Pizza Disaster

Starlight Middle School does not have enough skates for all of the students. The Student Council decides to sell pizza at lunch to raise money for extra skates. The personal size pizzas that they are selling are rectangular with dimensions of 4 inches by 6 inches. The perimeter of each pizza is decorated with toppings of sausage or mushrooms.

In order to make more money, Alex decides that making pizzas with the same shape but with dimensions that are twice the size, three times the size, and four times the size would bring in more money. Alex made the poster shown on the right.



Pizza Extravaganza	
Size	**Price**
Personal size	$2
Personal size × 2	$4
Personal size × 3	$6
Personal size × 4	$8

A. At the end of the day, Alex was totally confused. They had used up the week's supply of pizza dough and toppings, but they had not made that much more money. Mr. Hadley, the math teacher, took one look at the sign and shook his head. "Remember what you learned in math class this year?" He advised Alex to complete the table. Help Alex by finding the perimeter and area of each pizza size. Then find the scale factor of the bigger pizzas to the personal size pizza.

Size	Dimensions	Perimeter	Area	Scale Factor
Personal size				
Personal size × 2				
Personal size × 3				
Personal size × 4				

B. Alex realized that the personal size × 2 pizza was not twice as big as he had planned. Look at the table and discuss with your partner what you think went wrong with Alex's plan.

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Investigate Problem 2

1. How do the area and perimeter change compared to how the scale factor changes? Write your answer using a complete sentence.

2. How much greater is the area of the personal size × 3 pizza than the personal size pizza? How much greater is the area of the personal size × 4 pizza than the personal size pizza?

3. How many times greater is the perimeter of the personal size × 3 pizza than the personal size pizza? How many times greater is the perimeter of the personal size × 4 pizza than the personal size pizza?

4. How do the answers to Questions 2 and 3 relate to the scale factor? Write your answer using a complete sentence.

5. What dimensions could Alex have used to make a pizza with an area twice as large as the personal size pizza? Write your answer using a complete sentence.

6. What dimensions could Alex have used to make a pizza with an area three times as large as the personal size pizza? Write your answer using a complete sentence.

7. What dimensions could Alex have used to make a pizza with an area four times as large as the personal size pizza? Write your answer using a complete sentence.

8. The personal size pizza feeds one person. How many students should Alex's personal size × 2 pizza feed?

   How many students should Alex's personal size × 3 pizza feed?

   How many students should Alex's personal size × 4 pizza feed?

Investigate Problem 2

9. The personal size pizza feeds one person for $2. What prices should Alex have charged for each of the larger pizzas? Complete the poster with the new prices.

Pizza Extravaganza	
Size	**Price**
Personal size	$2
Personal size × 2	
Personal size × 3	
Personal size × 4	

10. Suppose that Alex had made a personal size × 10 pizza using his method from Part (A). How many times greater is the perimeter of the personal size × 10 pizza than the personal size pizza?

How many times greater is the area of the personal size × 10 pizza than the personal size pizza?

How many students would the personal size × 10 pizza feed?

11. Alex also thought he would make a pizza using his method from Part (A) but with dimensions that are one half of the size for dieters and charge $1. What would the dimensions of this personal size × $\frac{1}{2}$ pizza be?

What would the area of the personal size × $\frac{1}{2}$ pizza be?

What would the perimeter of the personal size × $\frac{1}{2}$ pizza be?

12. How do the area and perimeter of the personal size × $\frac{1}{2}$ pizza relate to the scale factor? Write your answer using a complete sentence.

13. Alex learned an important and costly lesson with the pizza disaster. Write several complete sentences explaining how the area and perimeter of scaled figures relate to the scale factor.

14. Complete the sentence: If two figures are similar and the scale factor is 5, the area of the larger figure is _____ times the area of the smaller figure and the perimeter of the larger figure is _____ times the perimeter of the smaller figure.

10.2

Round Food Around the World

Circumference and Area of a Circle

Objectives

In this lesson, you will:

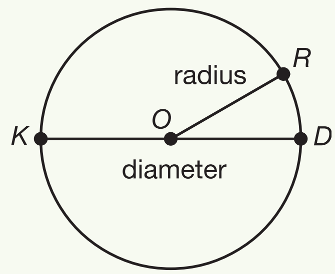
- Find circumferences of circles.
- Find areas of circles.

Key Terms

- circle
- center
- radius
- diameter
- circumference
- pi

Problem 1 Round Bread

A **circle** is a figure that consists of all points in a plane that are the same distance from a fixed point, called the **center** of the circle. The distance from the center to any point on the circle is the **radius** of the circle. The distance across the circle through the center is the **diameter** of the circle. The radius of a circle is one half of the diameter of the circle.



Point *O* is the center of the circle.

Segment *OR* is a radius of the circle.

Segment *KD* is a diameter of the circle.

The length of segment *KO* is one half of the length of segment *KD.*

In many countries of the world, bread is in the shape of a circle. Complete the table by finding the radius or the diameter of the type of bread.

Part of the World	Name of Bread	Radius	Diameter
Middle East	pita	5 inches	
Latin America	tortilla		8 inches
Ireland	soda bread	4 inches	
India	chapati		12 inches
Ethiopia	iniera	10 inches	

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Investigate Problem 1

1. In Lesson 10.1, we learned that the perimeter is the distance around a figure. The distance around a circle has a special name—the **circumference.**

Form a group with another partner team. Use string and a ruler or a tape measure to find the distance around several circular objects (the circumference). Then measure the distance across the object through the center (the diameter). Record your measurements in the table.

Object	**Circumference, *C***	**Diameter, *d***	**Ratio of Circumference to Diameter, $\frac{C}{d}$**

2. The ratio $\frac{C}{d}$ is called **pi** (pronounced "pie") and is written using the Greek letter π. Compare your results with those of your group members. Is the ratio $\frac{C}{d}$ close to the approximate value of π, which is 3.14? Write your answer using a complete sentence.

3. Press the π key on a calculator. Use a complete sentence to describe what the calculator shows.

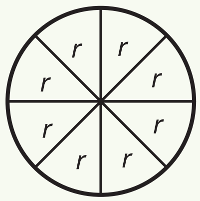
4. Write a formula for the circumference C of a circle that uses π and the diameter d.

5. Write a formula for the circumference C of a circle that uses π and the radius r.

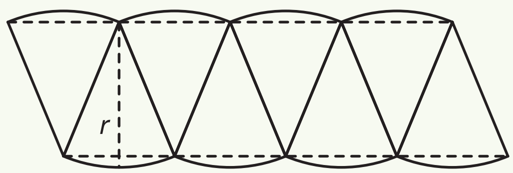
Problem 2 Making Strudel

A strudel is a German pastry that is made by first layering fruit between thin sheets of dough and then baking. You and your cousin are making strudel. You make the strudel in the shape of circle, but then decide that it should be in the shape of a rectangle. What can you do?

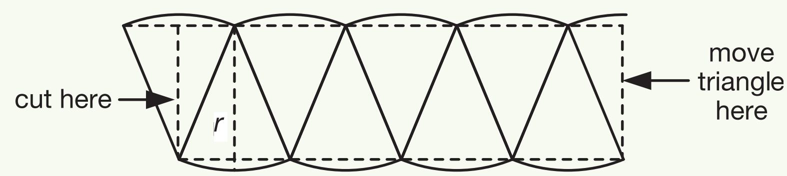
A. Each member of your group should create a diagram of the strudel by cutting a circle out of paper. Then divide the circle into eight or more pie-shaped congruent sections (sections of equal size and shape).



B. Cut out each pie-shaped section. The rearrange the sections into a single figure, as shown.



C. The figure that you formed is approximately a parallelogram. You can form a rectangle from the parallelogram.



D. So, you can cut the strudel and make a rectangle from a circle. We can use the formula for the area of a rectangle to develop the formula for the area of a circle. The radius is the height of the rectangle. The length of the rectangle is half of the circumference of the circle. Write a formula for the area of a circle using π and r. Use a complete sentence to explain how you found the formula.

E. Compare your formula with the formulas of others in your group.

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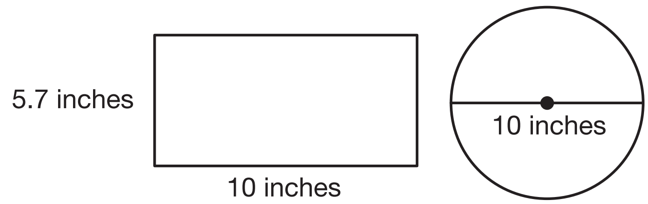
Investigate Problem 2

1. Two strudels are shown below. The perimeter of the rectangular strudel is about the same as the circumference of the round strudel. Which strudel has the greater area?



Area of rectangular strudel:

Area of circular strudel:

2. Collect the prices of small, medium, and large pizzas from two pizza restaurants. You will also need the diameters, in inches, of the pizzas. Use the information you collect to complete the tables below.

Size	Price	Diameter	Radius	Circumference	Area	Price per Square Inch
Small						
Medium						
Large						

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Size	Price	Diameter	Radius	Circumference	Area	Price per Square Inch
Small						
Medium						
Large						

3. For each restaurant, determine which size of pizza is the best buy. Use complete sentences to explain your reasoning.

10.3 City Planning

Areas of Parallelograms, Triangles, Trapezoids, and Composite Figures

Objectives

In this lesson, you will:

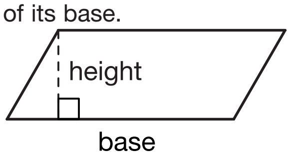
- Find areas of triangles, parallelograms, and trapezoids.
- Find areas of composite figures.

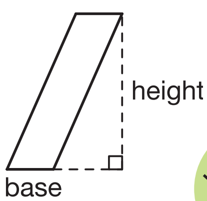
Key Terms

- composite figures

Take Note

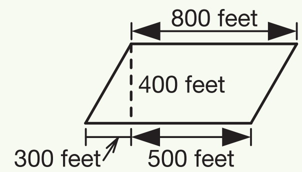
A parallelogram is described by its height and the length of its base.



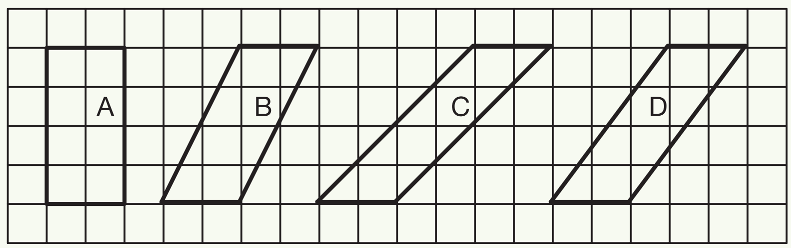


Problem 1 Areas of Parallelograms

In the city where Starlight Middle School is located, the city planners want to build a skating park. The shape of the park is a parallelogram, shown at the right. What is the area of the park?



A. Below is a family of parallelograms. Find the area of each parallelogram.



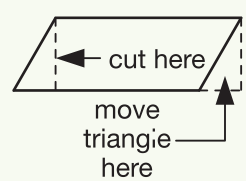
Area of parallelogram A: Area of parallelogram B:

Area of parallelogram C: Area of parallelogram D:

B. What patterns do you see in the areas of the family of parallelograms? Write a complete sentence to explain why they would be called a family.

C. Form a group with another partner team. Each person in your group should draw a parallelogram (that is not a rectangle) on a piece of grid paper. Record the base, height, area, and perimeter of your parallelogram. Cut your parallelogram in two pieces so that it can be reassembled into a rectangle. Record the length, width, perimeter, and area of the new rectangle that you formed.

Parallelogram	**Rectangle**
Base:	Length:
Height:	Width:
Area:	Area:
Perimeter:	Perimeter:



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Investigate Problem 1

1. In Part (C), what relationship do you see between the area of the rectangle and the area of the original parallelogram? Write a complete sentence to describe the relationship.

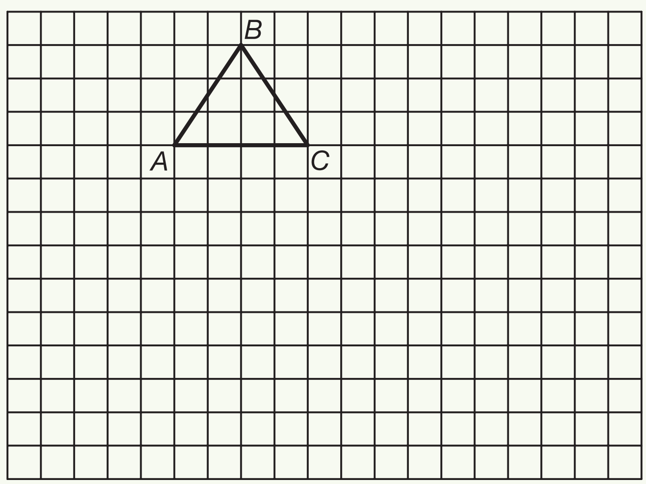
2. Use what you discovered to find the area of the park in Problem 1.

3. The measures of the length and width of the rectangle you formed are the same as the measures of the base and height of the original parallelogram. Write the formula for finding the area of any parallelogram.

4. Is the perimeter of the parallelogram the same as the perimeter of the rectangle in Part (C)? Use complete sentences to explain.

Problem 2 Areas of Triangles

A. On the grid below, draw 5 triangles that have the same area as triangle *ABC.*



B. Write the length of the base and the height of each triangle on the grid.

C. What do all of these triangles have in common besides their areas? Use complete sentences to explain.

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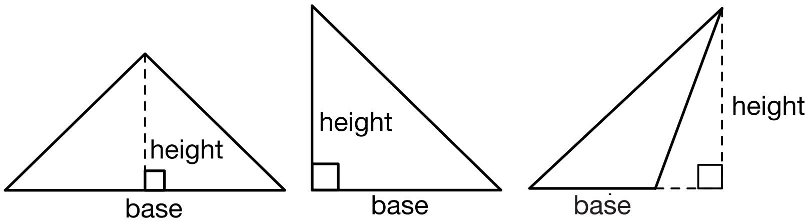
Investigate Problem 2

1. As with a parallelogram, a triangle is described by the length of its base and its height.



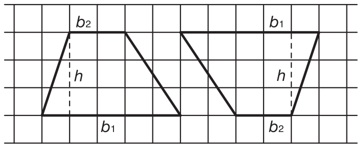
In your group, each person should draw a triangle on a sheet of grid paper. Record the base and height of the triangle. Cut out the triangle and trace it to make another copy of your triangle. Cut out the copy of the triangle. Piece together your two triangles to form a parallelogram. Record the base and height of the parallelogram.

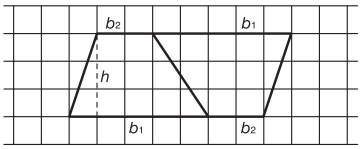
How do the length of the base and height of the parallelogram compare to those of the original triangle? Write your answer using complete sentences.

2. In your group, each person should draw and cut out a parallelogram. Then cut it into two congruent triangles. What conclusions can you make about the area of a triangle and the area of a parallelogram? Use a complete sentence to write your answer.

3. How do you think the formula for the area of a triangle is related to the formula for the area of a parallelogram? Use a complete sentence to write your answer.

4. In your group, each person should cut out two congruent trapezoids. Rearrange the trapezoids to form a parallelogram.





5. Use what you know about finding the area of a parallelogram to find the area of a trapezoid. Write the formula for the area of a trapezoid using the variables b_1, b_2, and h.

Take Note

You can think of the height of a triangle as being the distance that a rock would fall if you dropped it from the top vertex of the triangle down to a line through the base.

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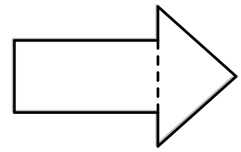
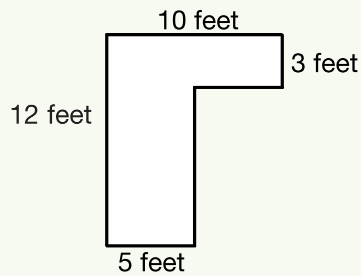
A **composite figure** is a figure that can be divided into several common figures. For instance, the composite figure at the right can be divided into a rectangle and a triangle.

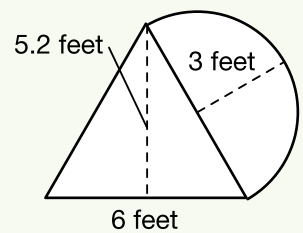


To find the area of a composite figure, find the area of each common figure that makes up the composite figure and then add the areas.

Problem 3 Areas of Composite Figures

A city planner has been given the task of designing several new and unique features to be placed throughout the city. The city wants to add several new flower gardens. Find the area of each new flower garden.





Area of first rectangle:

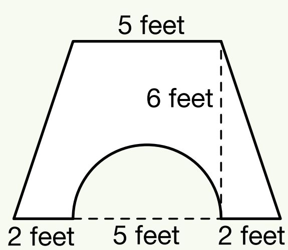
Area of second rectangle:

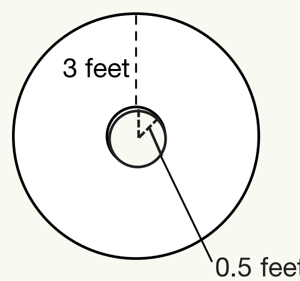
Area of composite figure:

Area of triangle:

Area of half of circle:

Area of composite figure:





Area of trapezoid:

Area of half of circle:

Area of composite figure:

Area of larger circle:

Area of smaller circle:

Area of composite figure:

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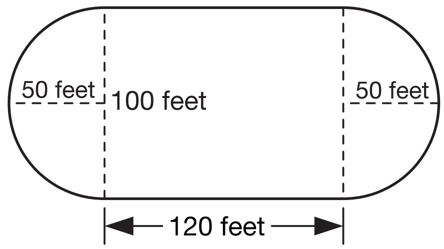
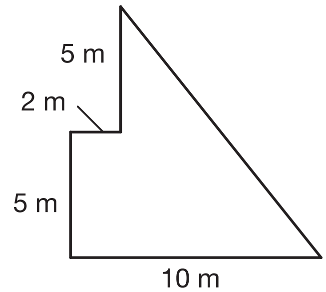
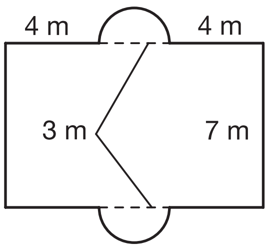
Investigate Problem 3

1. The city wants to build a swimming pool. The new design that uses half circles on the ends is shown below. Find the swimming pool's area.

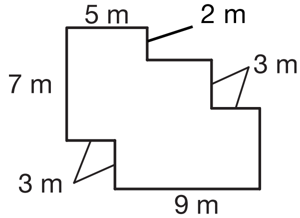


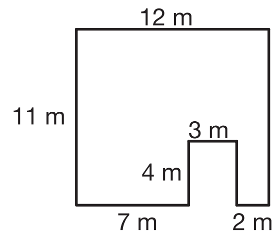
2. The city planner needs to design stages for two different parks in the city. Find the area of each stage.





3. The children of the city are excited because the city is planning two new playgrounds. Find the area of each playground.





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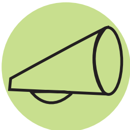

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10.4 Sports Fair and Square

Squares and Square Roots

Objectives

In this lesson, you will:

- Find squares of numbers.
- Find and estimate square roots of numbers.

Key Terms

- square
- perfect square
- square root
- radical sign
- radicand

Take Note

Taking the square root of a number is the *inverse operation* of squaring a number.

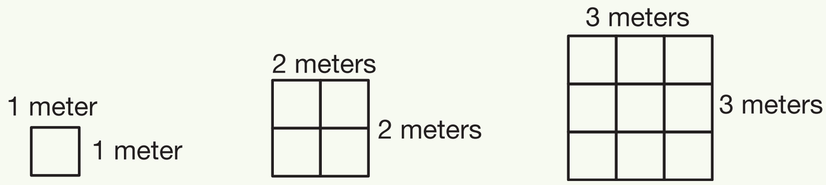
Take Note

The number underneath a radical symbol is called the **radicand.**

Problem 1 Judo Match

A judo match is held on a square mat that has an area of 676 square feet. How can you find the length of the side of the mat?

A. Find the area of each square below.



Area = ___ square meter   Area = ___ square meters   Area = ___ square meters

B. How does the area of each square relate to the length of its side? Write your answer using a complete sentence.

C. We can write this relationship as the **square** of a number:

$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$

Write the squares of the next seven whole numbers. These numbers are called **perfect squares.**

D. The factors that are multiplied to form a perfect square are called *square roots.* The **square root** of a number is one of the two identical factors of the number. Perfect squares have integers as their square roots. For instance, the square roots of 36 are 6 and –6 because (6)(6) = 36 and (–6)(–6) = 36. So, every number has two square roots, a positive square root and a negative square root. We write a square root using the **radical sign** $\sqrt{}$.
A radical sign indicates the positive square root of a number.

$\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$ $\sqrt{16} = 4$

Write the square root of each perfect square.

$\sqrt{121} =$ $\sqrt{169} =$ $\sqrt{400} =$

E. The square root of 676 is the length of the side of the karate mat. Find $\sqrt{676}$ to find the length of the side of the mat.

10

Investigate Problem 1

Take Note

You can write the square roots of 36 as ±6, meaning positive 6 and negative 6. The positive square root, 6, is called the *principal square root* of 36.

1. Most numbers do not have integers for their square roots. To estimate the (positive) square root of a number that is not a perfect square, begin by finding the two perfect squares closest to the number so that one is less than the number and one is greater than the number. Then use trial and error to find the best estimate for the square root.

Estimate $\sqrt{10}$ to the nearest tenth.

Nine is the closest perfect square less than 10 and 16 is the closest perfect square greater than 10.

So, $\sqrt{10}$ is between $\sqrt{9}$ = ____ and $\sqrt{16}$ = ____. Now estimate the square root to the nearest tenth by choosing numbers between 3 and 4 and finding the square of these numbers to determine which one is the best estimate.

(3.1)(3.1) = ______ (3.2)(3.2) = ______

Which number's square is closer to 10?

So, $\sqrt{10} \approx$ ____. The symbol ≈ means **"approximately equal to."**

2. Estimate $\sqrt{39}$ to the nearest tenth.

____ is a perfect square less than 39 and ____ is a perfect square greater than 39.

So, $\sqrt{39}$ is between ________ and ________.

$\sqrt{39} \approx$ ____

3. Estimate $\sqrt{27}$ to the nearest tenth.

____ is a perfect square less than 27 and ____ is a perfect square greater than 27.

So, $\sqrt{27}$ is between _________ and ________.

$\sqrt{27} \approx$ ____

4. Estimate $\sqrt{60}$ to the nearest tenth.

____ is a perfect square less than 60 and ____ is a perfect square greater than 60.

So, $\sqrt{60}$ is between ________ and ________.

$\sqrt{60} \approx$ ____

Investigate Problem 1

5. For each number, race your partner to estimate the square root of the number. You may use a calculator, but you may not use the calculator's square root key. When you complete a number, see whether you or your partner has a closer estimate by squaring each of your estimates.

$\sqrt{79} \approx$

$\sqrt{135} \approx$

$\sqrt{2} \approx$

$\sqrt{30} \approx$

6. Explain how you found your estimates to your partner.

7. Why don't negative numbers have square roots? Use a complete sentence to explain your answer.

Problem 2 Square Sports Arenas

Each sport below is played on a square mat. Find the dimensions of each mat given its area. Write your answer using a complete sentence.

Gymnastics

1600 square feet

Wrestling

1521 square feet

Boxing

484 square feet

Judo

576 square feet

10

Investigate Problem 2

1. Find the area of a square whose perimeter is 40 feet. Use complete sentence to explain how your found the area.

2. Find the area of a square whose perimeter is 64 inches. Write your answer using a complete sentence.

3. Find the perimeter of a square whose area is 81 square meters. Use complete sentences to explain how you found the perimeter.

4. Find the perimeter of a square whose area is 256 square meters. Write your answer using a complete sentence.

5. For a judo competition, an instructor needs to create two square competition areas. She needs to purchase 1089 square feet of tatami mats to cover the floor of Area A and 184 feet of tape to mark the outside edge of the Area B. Which competition area is bigger? Use complete sentences to explain how you determined the bigger area.

6. Use the square root key of your calculator to find the square root of each number to the nearest hundredth.

$\sqrt{5} \approx$ $\sqrt{7} \approx$

$\sqrt{19} \approx$ $\sqrt{32} \approx$

$\sqrt{56} \approx$ $\sqrt{78} \approx$

10.5 Are You Sure It's Square?

The Pythagorean Theorem

Objectives

In this lesson, you will:

- Prove the Pythagorean theorem.
- Use the Pythagorean theorem to solve problems.

Key Terms

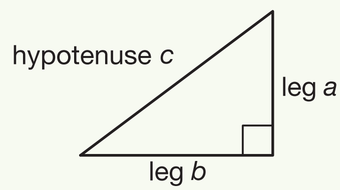
- leg
- hypotenuse
- Pythagorean theorem

The Pythagorean theorem is one of the best known theorems in mathematics. It is named for Pythagoras, a Greek mathematician that lived in approximately 500 B.C. Although Pythagoras is given credit for discovering the theorem, the ancient Babylonians, Chinese, and Egyptians understood it and used it to construct their buildings and land boundaries.

Problem 1 Is It Square?

Your family is building a rectangular cabin that has a length of 40 feet and a width of 30 feet. You need to make sure to set the posts at the four corners. How do you know that the cabin is in the shape of a rectangle and not a parallelogram?

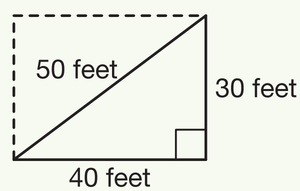
A rectangle can be divided into two right triangles. The sides that form the right angle are called the **legs.** The side opposite of the right angle is the **hypotenuse.**



The **Pythagorean theorem** is a formula that relates the lengths of the three sides of a right triangle. It states that if *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse, then

$a^2 + b^2 = c^2$.

You measure the hypotenuse of your cabin and find it to be 50 feet in length. Fill in the boxes to show that the Pythagorean theorem is true for the dimensions of your cabin.



$$a^2 + b^2 = c^2$$

$$\square^2 + \square^2 = \square^2$$

$$\square + \square = \square$$

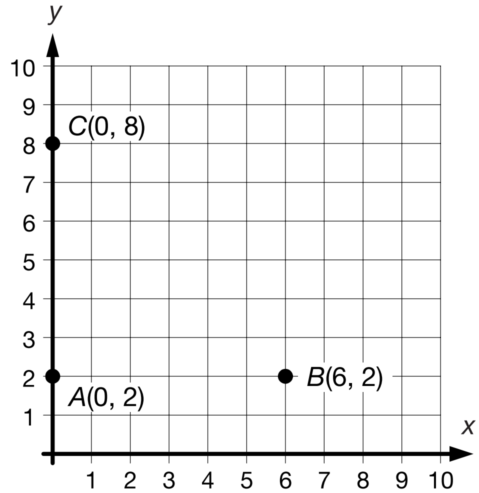
$$\square = \square$$

Write the Pythagorean theorem in your own words.

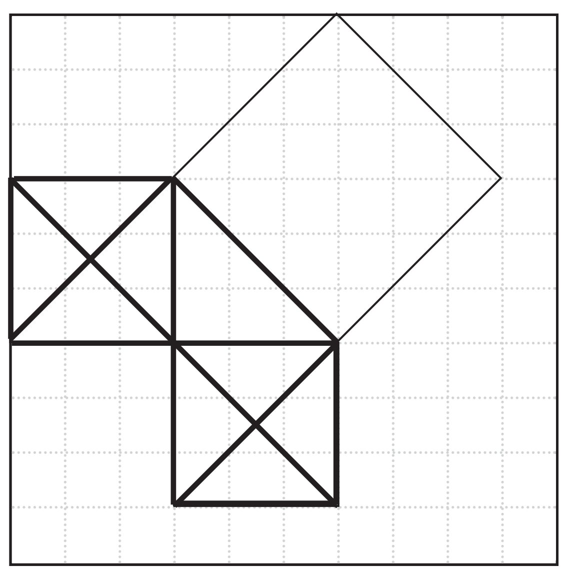
10

Investigate Problem 1

1. Use the Pythagorean theorem to find the distance from point *C* to point *B* in the coordinate plane. Use a complete sentence to explain how you found the distance.



2. There is a special relationship that exists between the squares of the lengths of the sides of a right triangle. Use graph paper to draw an isosceles right triangle. Draw squares on each side of the triangle as shown. Then in each of the two smaller squares, draw the diagonals. Cut out the two smaller squares. Then cut those squares into fourths along the diagonals. Fit these pieces on top of the larger square.



Take Note

Recall that an isosceles right triangle is a right triangle whose legs have the same length.

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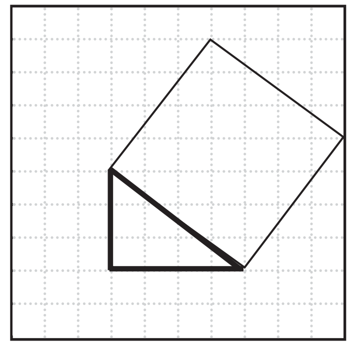
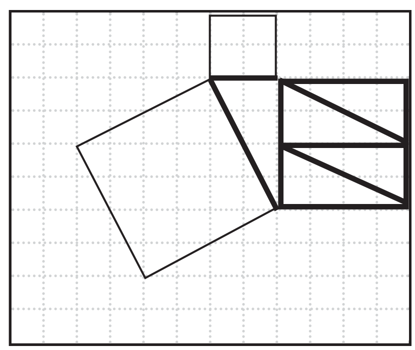
Investigate Problem 1

3. On graph paper, draw a right triangle that has one leg that is 3 units in length and one leg that is 4 units in length. Draw a square on the hypotenuse of the triangle as shown. Cut out a 3-unit by 3-unit square and a 4-unit by 4-unit square from the same graph paper. Then cut the two squares into strips that are either 4 units by 1 unit or 3 units by 1 unit, and individual squares of 1 unit by 1 unit. Arrange the strips and squares on top of the square along the hypotenuse.



What is the relationship between the area of the squares you cut out and the area of the square you drew? Use a complete sentence to write your answer.

4. On graph paper, draw a right triangle so that the length of one leg is twice the length of the other leg. Draw a square along each leg of the triangle as shown. Cut out the smaller square. Then cut out the larger square and cut it into four congruent triangles as shown. Draw a square along the hypotenuse. Arrange the smaller square and the triangles to exactly cover the square that lies along the hypotenuse.



Does the special relationship between the legs and the hypotenuse hold? Use a complete sentence to explain.

5. Does the Pythagorean theorem work for triangles that are not right triangles? Use complete sentences to explain your reasoning.

10

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10.6 A Week at Summer Camp
Using the Pythagorean Theorem

Objectives

In this lesson, you will:

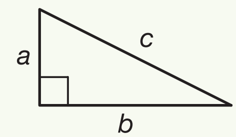
- Use the converse of the Pythagorean theorem.
- Find Pythagorean triples.

Key Terms

- converse
- Pythagorean triple

Problem 1 Morning of Day One

In Lesson 10.5, we learned the Pythagorean theorem:

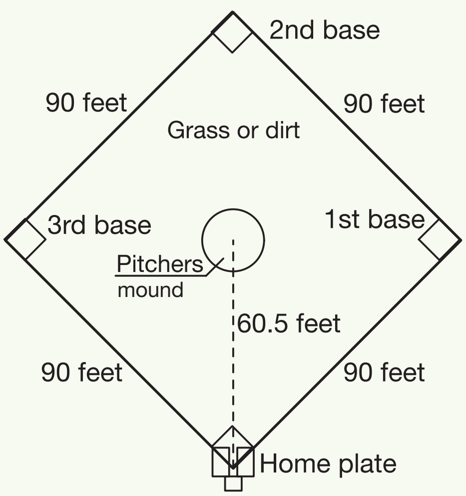


If **a triangle is a right triangle,** then $\boldsymbol{a^2 + b^2 = c^2}$.

The **converse** of a theorem is created when the if-then parts of the theorem are exchanged. Switch the if-then parts of the Pythagorean theorem and complete the statement below, which is the converse of the Pythagorean theorem.

If _______________, then **the triangle is a** ____________ **triangle.**

A. In the morning of Day 1 at summer camp, you are playing a game of baseball. The catcher has a chance to get a player out who is trying to steal second base. How far must the catcher throw the ball if she is on home plate throwing to the player on second base? Write your answer using a complete sentence.



B. Did you use the Pythagorean theorem or its converse to solve the problem? Use complete sentences to explain.

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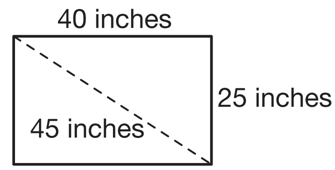
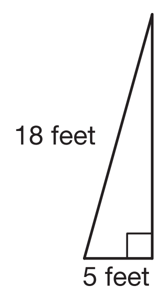
Investigate Problem 1

1. In the afternoon of Day 1 at summer camp, you are scheduled for woodworking class. You are making a table with a length of 40 inches and a width of 25 inches. You measure the diagonal of the table to be 45 inches. Is the table rectangular?
That is, does the table have corners that are right angles?
Write your answer using a complete sentence.



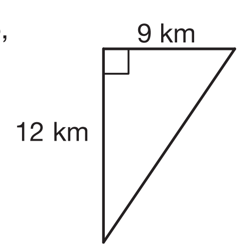
2. Did you use the Pythagorean theorem or its converse to solve Question 1? Write a complete sentence to explain.

3. In the morning of Day 2 at summer camp, you are helping wash the windows of your cabin. You have an 18-foot ladder that you place 5 feet from the base of the cabin wall. How far up the wall will the top of the ladder be? Round your answer to two decimal places. Write your answer using a complete sentence.



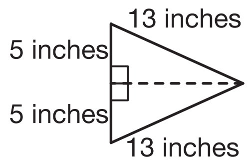
4. Write a complete sentence explaining how you answered Question 3 using the Pythagorean theorem.

5. In the afternoon of Day 2 at summer camp, you take a hike, walking 12 kilometers north and then 9 kilometers east. How many kilometers must you hike to get back to your starting point along the path shown?

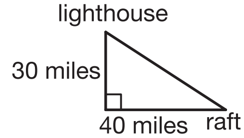


Investigate Problem 1

6. In the morning of Day 3 at summer camp, you use material to make camp pennants. How many square inches of material did you use to make your pennant, shown at the right? Write your answer using a complete sentence.

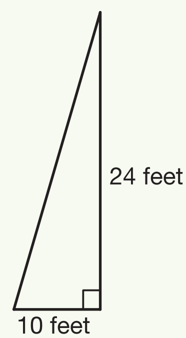


7. In the afternoon of Day 3 at summer camp, you take a rafting trip. When you are 40 miles from camp on your raft, how far are you from the lighthouse? Use the map at the right to help you. Write your answer using a complete sentence.



Problem 2 *Evening of Day Three*

A. When you return from your rafting trip, you find that you have locked yourself out of your cabin. The only open window is on the second floor 24 feet above the ground. You find an adjustable ladder nearby, but there's a bush along the edge of the cabin, so you have to place the ladder 10 feet from the cabin. What length of ladder do you need to reach the window? Write your answer using a complete sentence.



B. Write the Pythagorean theorem as it relates to the situation in Part (A). What do you notice about the lengths of the sides of the right triangle shown?

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Investigate Problem 2

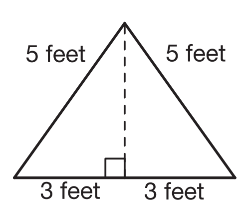
1. **Math Path: Pythagorean Triples**

   You may have noticed in part (B) of Problem 2 that all of the side lengths of the right triangle were integers. The set of three positive integers *a, b,* and *c* that satisfy the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple.** For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$. Form a group with another partner team. Find as many Pythagorean triples as you can.

2. Each group should take turns sharing their Pythagorean triples with the rest of the class. Keep a class list on the board or some other space that everyone in the class can see. Look for patterns in the list. Use a complete sentence to explain what you notice.

3. In the afternoon of Day 4 at summer camp, you ride a bicycle 8 miles north and then 5 miles west. Draw a diagram to represent the situation. What is the shortest distance to the nearest tenth of a mile that you must travel to return to the camp? Write your answer using a complete sentence.

4. In the evening of Day 4 at summer camp, you plan to sleep out in a tent. The tents are triangular in shape and are constructed so that the cloth that forms the tent is 10 feet long. The bottom of the tent is 6 feet wide. What is the height of the pole that you need to hold the tent in place? Write your answer using a complete sentence.



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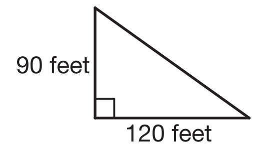
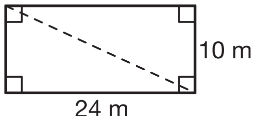
Investigate Problem 2

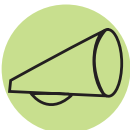
5. In the morning of Day 5 at summer camp, you help your camp counselor move a large round table into the dining hall. The door to the dining hall is 8 feet high and 3 feet wide. The table measures 8.5 feet in diameter. Your counselor decides that it is impossible to fit the table through the door, so he gives up. Explain to him how it is possible to move the table through the door. Draw a diagram to represent the situation. Write your answer using complete sentences.

6. In the afternoon of Day 5 at summer camp, you watch several workers at the camp construct a new ramp for water skiing on the lake. You take a peek at the blueprint for the ramp, shown at the right. How long must the workers make the ramp? Write your answer using a complete sentence.



7. On the morning of the last day at camp, your grandmother comes to pick you up. You want to show her how much your swimming has improved, so you swim across the camp pool diagonally 10 times. A diagram of the pool is shown at the right. How far do you swim to impress your grandmother? Write your answer using a complete sentence.





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Looking Back to Chapter 10

Key Terms

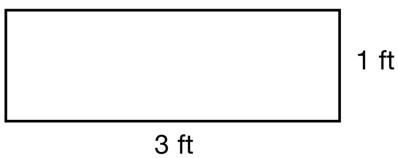
perimeter ● p. 305
area ● p. 305
circle ● p. 311
center ● p. 311
radius ● p. 311
diameter ● p. 311
circumference ● p. 312
pi ● p. 312
composite figure ● p. 318
square ● p. 321
perfect square ● p. 321
square root ● p. 321
radical sign ● p. 321
radicand ● p. 321
leg ● p. 325
hypotenuse ● p. 325
Pythagorean theorem ● p. 325
converse ● p. 329
Pythagorean triple ● p. 332

Summary

Finding Perimeters of Rectangles (p. 305)

To find the perimeter of a rectangle, find the distance around the rectangle. You can also multiply 2 by the rectangle's length and 2 by the rectangle's width, and add the products.

Example



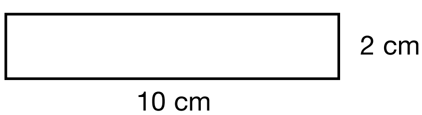
Perimeter = 3 + 1 + 3 + 1 = 8 feet

Perimeter = $2\ell + 2w = 2(3) + 2(1) = 6 + 2 = 8$ feet

Finding Areas of Rectangles (p. 305)

To find the area of a rectangle, multiply the length of the rectangle by the width of the rectangle.

Example



Area = $\ell \times w$

= 10×2

= 20 square centimeters

Finding Circumferences of Circles (p. 312)

To find the circumference of a circle, multiply the radius of the circle by 2π. You can use 3.14 for π.

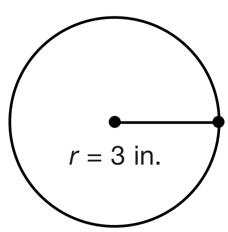
Example

Circumference = $2\pi r$

Circumference = $2\pi(3)$

$\approx 2(3.14)(3)$

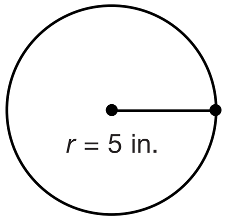
= 18.84 inches



Finding Areas of Circles (p. 313)

To find the area of a circle, multiply the square of the radius by π.

Example



Area = πr^2

Area = $\pi(5)^2$

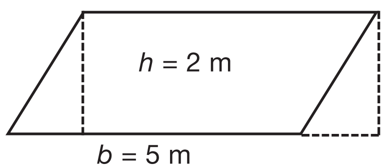
$\approx (3.14)(25)$

$= 78.5$ square inches

Finding Areas of Parallelograms (p. 315)

To find the area of a parallelogram, multiply the height of the parallelogram by the length of its base.

Example



Area = bh

Area = $5(2)$

$= 10$ square meters

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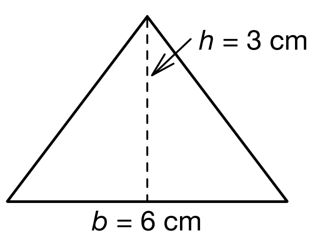
Finding Areas of Triangles (p. 317)

To find the area of a triangle, multiply $\frac{1}{2}$ of the base of the triangle by the triangle's height.

Example



Area = $\frac{1}{2}bh$

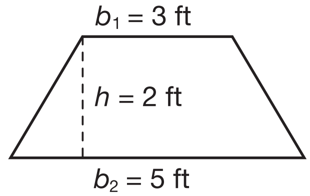
Area = $\frac{1}{2}(6)(3)$

$= 9$ square centimeters

Finding Areas of Trapezoids (p. 317)

To find the area of a trapezoid, multiply $\frac{1}{2}$ of the sum of the bases of the trapezoid by the height of the trapezoid.

Example



$$\text{Area} = \frac{1}{2}(b_1 + b_2)h$$

$$\text{Area} = \frac{1}{2}(3 + 5)(2)$$

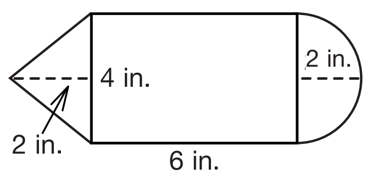
$$= \frac{1}{2}(8)(2)$$

$$= 8 \text{ square feet}$$

Finding Areas of Composite Figures (p. 318)

To find the area of a composite figure, find the area of each common figure that makes up the composite figure and then add the areas.

Example



Area of triangle = $\frac{1}{2}(4)(2) = 4$ square inches

Area of rectangle = $4 \times 6 = 24$ square inches

Area of half of circle = $\frac{1}{2}(\pi)(2^2) \approx \frac{1}{2}(3.14)(4) = 6.28$ square inches

Area of composite figure = $4 + 24 + 6.28 = 34.28$ square inches

Finding Squares of Numbers (p. 321)

To find the square of a number, multiply the number by itself.

Example

To find the square of 3, multiply 3 by itself.

$3 \times 3 = 3^2 = 9$

So, the square of 3 is 9.

10

Finding Square Roots of Numbers (p. 321)

To find the square roots of a number that is a perfect square, find the identical factors of the number. One factor is the square root.

Example The number 49 is a perfect square. To find the square roots of 49, you know that (7)(7) = 49 and that (–7)(–7) = 49. So, the square roots of 49 are 7 and –7.

The positive square root is $\sqrt{49} = 7$.

To estimate the positive square root of a number that is not a perfect square, begin by finding two perfect squares, one less than the number and one greater than the number. Then use trial and error to estimate the square root.

Example To estimate $\sqrt{18}$, you know that 16 is the closest perfect square less than 18 and 25 is the closest perfect square greater than 18. So, $\sqrt{18}$ is between $\sqrt{16} = 4$ and $\sqrt{25} = 5$.

(4.1)(4.1) = 16.81

(4.2)(4.2) = 17.64

(4.3)(4.3) = 18.49

So, $\sqrt{18}$ is between 4.2 and 4.3.

You can estimate $\sqrt{18} \approx 4.25$.

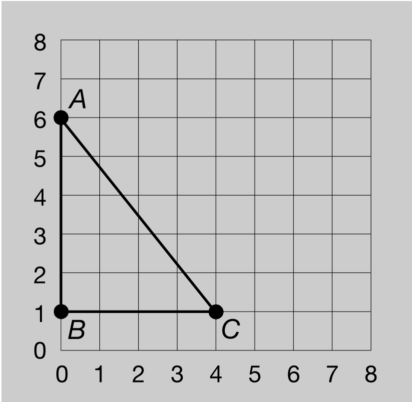
Using the Pythagorean Theorem (p. 325)

The Pythagorean theorem states that if a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Example Triangle ABC is a right triangle. To find the distance from point A to point C, you can use the Pythagorean theorem.

The distance from A to B is 5 units.

The distance from B to C is 4 units.



Let the variable c represent the distance from A to C. Then substitute values into the Pythagorean theorem.

$5^2 + 4^2 = c^2$

$41 = c^2$

$c = \sqrt{41} \approx 6.4$

So, the distance from point A to point C is about 6.4 units.

You can use the converse of the Pythagorean theorem to show that a triangle is a right triangle. If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

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Finding Pythagorean Triples (p. 332)

To find a Pythagorean triple, find any three integers that satisfy the equation $a^2 + b^2 = c^2$.

Examples The integers 5, 12, and 13 are a Pythagorean triple because $5^2 + 12^2 = 13^2$.

The integers 20, 21, and 29 are a Pythagorean triple because $20^2 + 21^2 = 29^2$.