Looking Ahead to Chapter 7

7

Focus In Chapter 7, you will work with integers. You will add, subtract, multiply, and divide integers. You will also work with number lines, absolute value, and scientific notation.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 7.

Use mental math to find the product or quotient.

1. 3×9

2. 6×7

3. 5×8

4. $38 \div 2$

5. $45 \div 9$

6. $18 \div 6$

Write the prime factorization of the number.

7. 48

8. 64

9. 56

Read the problem scenario below.

Perry got 93 out of 100 problems correct on his math test. About half of the class scored above 82%.

10. What percent did Perry get on his math test?

11. Was his percent higher or lower than 82%? By how much was his percent higher or lower than 82%?

Key Terms

integer • p. 199
negative integer • p. 199
positive integer • p. 199
number line • p. 199
profit • p. 201
loss • p. 201
sum • p. 204
integer addition • p. 206
difference • p. 209
integer subtraction • p. 210
product • p. 211
quotient • p. 213
absolute value • p. 215
opposites • p. 217
additive inverse • p. 217
power • p. 219
exponent • p. 219
power of ten • p. 219
expanded form • p. 219
scientific notation • p. 223
negative exponent • p. 224

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Integers

The cent, once a copper coin, is now composed of copper-plated zinc that weighs 2.5 grams for each cent, about 20% less than an older penny. In Lesson 7.5, you will determine how far the mass of a penny is from specification.

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7.1 I Love New York

Negative Numbers in the Real World

Objectives

In this lesson, you will:

- Write integers to represent real-life situations.
- Graph integers on a number line.
- Compare integers.

Key Terms

- integer
- negative integer
- positive integer
- number line
- profit
- loss

One of the most important numbers in mathematics is the number zero. It was not until the Middle Ages that people needed a number for “no items.” After the concept of zero became widely understood, some mathematicians began to explore numbers less than zero.

In the 1800s, a well-defined system of integers was developed that included consistent rules for addition, subtraction, multiplication, and division. The numbers . . . , –4, –3, –2, –1, 0, 1, 2, 3, 4, . . . are **integers.** Integers include **negative integers** (integers less than zero), zero, and **positive integers** (integers greater than zero). In many situations, you can solve problems more easily by being able to represent quantities using integers.

Take Note

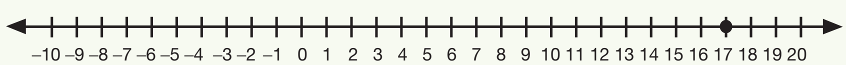
If a number (other than 0) has no sign, it is a positive integer. You read the integer 10 as “positive ten” instead of just “ten.” You read the integer –2 as “negative two.”

Problem 1 New York Highs and Lows

Your friend and her family are moving to New York City. She wants to know how cold it gets in the winter there. You help her by finding out the average low temperatures for the winter months.

Month	Dec.	Jan.	Feb.	Mar.
Average low temperature	17°F	–4°F	–2°F	10°F

A. A **number line** is a line that extends in both directions forever with one point that is assigned a value of zero and a given length assigned as one unit. Positive integers are assigned to the units on the right of zero and negative integers to the units on the left of zero. Use the number line below to graph each integer in the table. December is done for you.



B. On the number line, the values of integers increase as you move from left to right. Write the temperatures in order from least to greatest.

C. Use the number line to complete each statement. Use the symbol > for greater than and the symbol < for less than.

–1 ◯ –2   –9 ◯ –5   –3 ◯ 2

12 ◯ –8   0 ◯ –2   7 ◯ 0

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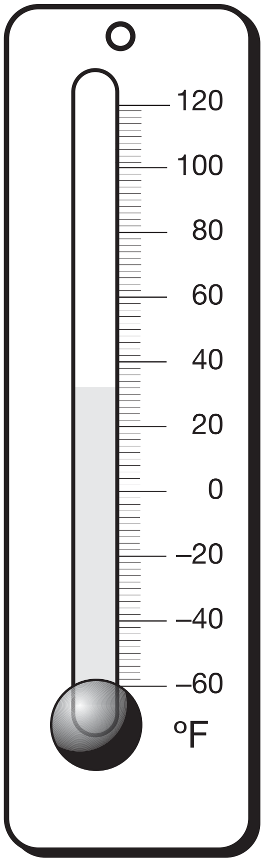

Investigate Problem 1

1. Write each temperature in degrees Fahrenheit as an integer. Use negative integers when necessary.

   Temperature when water freezes: _____

   Temperature outside today: _____

   Hottest temperature last summer: _____

2. In northern Alaska in January, it is sometimes as cold as 40 degrees below zero (in degrees Fahrenheit). On the same day in Death Valley, California, the temperature can be 95 degrees above zero (in degrees Fahrenheit). Write each temperature as an integer. Use the thermometer at the left to find the number of degrees between these temperatures.

3. The highest temperature recorded in New York was 108 degrees above zero (in degrees Fahrenheit) on July 22, 1926, at Troy. The lowest temperature recorded in New York was 52 degrees below zero (in degrees Fahrenheit), recorded on February 18, 1979, at Old Forge. Write each temperature as an integer. Then use the thermometer at the left to find the number of degrees between the temperatures.

Problem 2 On Wall Street

The New York Stock Exchange (NYSE) is located on Wall Street in New York City. At the NYSE, one measure of how well stocks are doing is the Dow Jones Industrial Average. You may have heard on the radio, "The Dow Jones Industrial Average lost 44 points today." Points are the units used to measure the combined gains and losses of stocks.

You can represent a gain in the Dow Jones Industrial Average as a positive integer and a loss as a negative integer. Complete the table by writing each gain or loss as an integer.

Dow Jones Industrial Average		
Date	**Gain or Loss**	**Gain or Loss as an Integer**
April 26, 2005	loss of 91 points	
April 27, 2005	gain of 47 points	
April 28, 2005	loss of 128 points	

Investigate Problem 2

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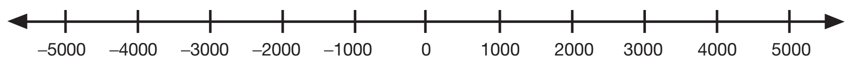
1. The value of a company's stock depends on many factors. One factor is whether a company makes a profit or a loss during a given time period. **Profit** is the amount of money that a company earns after expenses have been subtracted. **Loss** is the amount of money that a company loses because it does not earn enough money to cover its expenses. A profit is written as positive integer and a loss is written as a negative integer. Work with your partner to write each profit or loss as an integer.

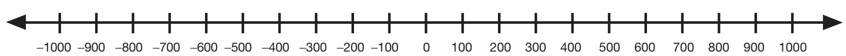
   A profit of \$400: ________ A loss of \$234: _________

   A loss of \$679,000: ________ A profit of \$560: ________

2. A company makes a profit of \$5000 one month and a loss of \$1000 the next month. Write each amount as an integer. Then plot each integer on the number line below. How many dollars are between the profit and the loss? Use the number line to help you.



3. Another company had a loss of \$500 the first week of the month, a loss of \$400 the second week, a profit of \$1000 the third week, and a loss of \$100 the last week of the month. Write each amount as an integer. Plot the integer representing the first week's loss on the number line below.



   Start at the point you just plotted and move to the left on the number line to represent a loss of \$400. At what point are you now on the number line?

   Next, move from the second point you plotted to the right to represent a profit of \$1000. At what point are you now on the number line?

   Finally, move from the third point you plotted to the left to represent a loss of \$100. At what point are you now on the number line?

   What is the company's total profit or loss for the month? Use a complete sentence to write your answer.

4. Share your answers with another partner team.

Problem 3 New York Heights and Depths

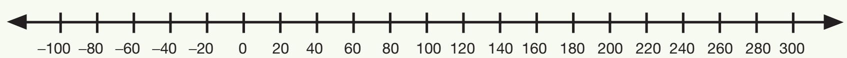
Integers are used to represent height (as the distance above the ground, water, or sea level) and depth (as the distance below the ground, water, or sea level). Height is written as a positive integer and depth is written as a negative integer.

A. Write each height or depth as an integer.

The Hudson River is about 45 feet deep.

The highest point in New York rises 5344 feet above sea level.

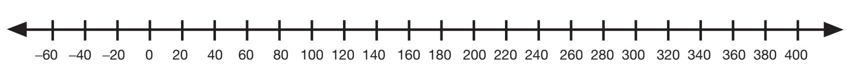
B. The Holland Tunnel in New York City is a tunnel that connects the island of Manhattan with New Jersey under the Hudson River. The deepest point of the tunnel is about 90 feet below sea level. The highest point in Manhattan, Bennett Park, is about 270 feet above sea level. Write the lowest point of the Holland Tunnel and the highest point in Manhattan as integers. Then plot each integer on the number line below.



C. Suppose you drive from Bennett Park through the tunnel. How many feet are between the highest point that you drove from and the lowest point that you drove through? Use the number line to help you.

Investigate Problem 3

1. You take an elevator up to the 86th Floor Observatory of the Empire State Building in New York City, which is 320 meters above street level. The 102nd Floor Tower, although closed to the public, is about 370 meters above street level. The basement of the building is about 10 meters below street level. The street level is at zero meters. Write the height of the 86th Floor Observatory, the height of the 102nd Floor Tower, and the depth of the basement, and the street level as integers. Then plot each integer on the number line below.



2. Suppose that you could take an elevator from the basement to the 102nd Floor Tower. How many meters are between the highest point that you were in the elevator and the lowest point? Use the number line to help you. Share your answers with another partner team.

7.2 Going Up?

Adding Integers

Objectives

In this lesson, you will:

- Add integers.

Key Terms

- sum
- integer addition

Problem 1 To the Top Floor

A large hotel has 26 floors of guest rooms above street level and 5 floors of parking below street level. The hotel's elevator can stop at every floor. Work with your partner to draw a diagram of the hotel's elevator.

Use your diagram to answer the following questions.

Suppose that the elevator starts at street level, goes up 7 floors, and then goes down 3 floors. On which floor would the elevator be?

Suppose that the elevator starts at street level, goes up 10 floors, and then goes down 12 floors. On which floor would the elevator be?

Suppose that the elevator starts at street level, goes down 4 floors, and then goes up 11 floors. On which floor would the elevator be?

Suppose that the elevator starts at street level, goes down 2 floors, then goes up 5 floors, and finally goes down 3 floors. On which floor would the elevator be?

Investigate Problem 1

1. We can assign positive integers to the floors above street level and negative integers to the floors below street level. Write an integer addition problem that models the elevator's motion in each case below.

   Starts at street level, goes up 7 floors, and then goes down 3 floors.

   Starts at street level, goes up 10 floors, and then goes down 12 floors.

   Starts at street level, goes down 4 floors, and then goes up 11 floors.

   Starts at street level, goes down 2 floors, then goes up 5 floors, and finally goes down 3 floors.

2. Use your diagram of the elevator to help you write a sentence that describes the motion of the elevator modeled by each integer addition problem below. Then find the sum to determine on which floor the elevator stops.

   $(-2) + 20 =$

   $12 + (-7) =$

   $2 + (-5) =$

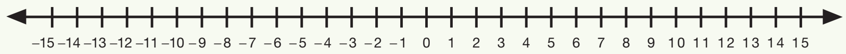
   $26 + (-20) + (-5) + (-3) =$

3. Join your group with another. Compare the sums that you found in Question 2 with others in your group. Then use complete sentences to explain whether the elevator motion model helped you understand the addition of integers.

Problem 2 *Using a Number Line to Add Integers*

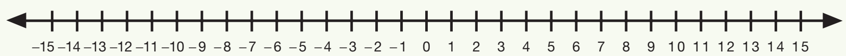
In Problem 1, we used the real-life representation of elevator motion to model integer addition. We can also use an abstract representation, the number line, to model integer addition.

A. Start at 5. Then move 4 units to the right. Represent this by graphing 5 on the number line and then drawing an arrow that starts at 5 and ends 4 units to the right of 5. Where are you on the number line?



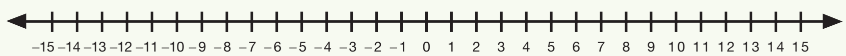
Write and solve an integer addition problem that represents this situation.

B. Start at –6. Then move 5 units to the right. Represent this by graphing –6 on the number line and then by drawing an arrow that starts at –6 and ends 5 units to the right of –6. Where are you on the number line?



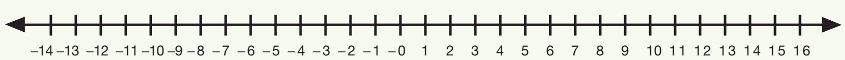
Write and solve an integer addition problem that represents this situation.

C. Start at –2. Then move 5 units to the left. Represent this by graphing –2 on the number line and then by drawing an arrow that starts at –2 and ends 5 units to the left of –2. Where are you on the number line?



Write and solve an integer addition problem that represents this situation.

D. Start at 12. Then move 4 units to the right. Then move 8 units to the left. Represent this by graphing 12 on the number line, then by drawing an arrow that starts at 12 and ends 4 units to the right of 12, and then by drawing arrow 8 units to the left of the previous ending point. Where are you on the number line?



Write and solve an integer addition problem that represents this situation.

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Investigate Problem 2

1. For each addition problem, find the sum. Then write a sentence that describes the movement on the number line that you could use to solve the problem.

(–12) + 15 =

8 + (–13) =

(–240) + 300 =

2450 + (–1500) =

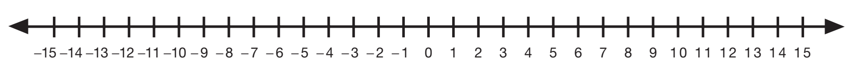
2. Math Path: Integer Addition

Complete the rule below for using a number line to add integers.

On a number line, move to the ___________ when you add a positive integer, and move to the ___________ when you add a negative integer.

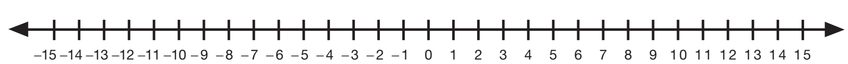
Use a number line to find each sum.

–14 + 1 =

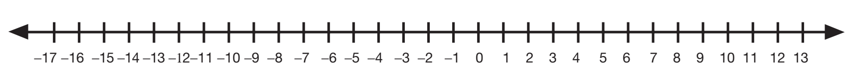


–11 + 11 =

9 + (–7) =



–8 + (–8) =



7.3 Test Scores, Grades, and More

Subtracting Integers

Objectives

In this lesson, you will:

- Subtract integers.

Key Terms

- difference
- integer subtraction

Problem 1 Correct and Incorrect?

A. On some tests, your final score is found by subtracting the number of incorrect responses from the number of correct responses. For example, if you answered 45 questions correctly and 10 questions incorrectly, your final score would be 35.

How can you represent the number of correct answers using positive integers? Use a complete sentence in your answer.

How can you represent the number of incorrect answers using negative integers? Use a complete sentence in your answer.

B. Work with your partner to answer each question.

Your teacher scores a test that has 30 correct answers and 30 incorrect answers. Write a subtraction problem that models this situation. Then write an integer addition problem that models this situation.

Your teacher scores a test that has 43 correct answers and 27 incorrect answers. Write a subtraction problem that models this situation. Then write an integer addition problem that models this situation.

Your teacher scores a test that has 10 correct answers and 15 incorrect answers. Write a subtraction problem that models this situation. Then write an integer addition problem that models this situation.

Your teacher scores a test that has 22 correct answers and 37 incorrect answers. Write a subtraction problem that models this situation. Then write an integer addition problem that models this situation.

Investigate Problem 1

1. Write a sentence that describes a test that could be modeled by each pair of subtraction and integer addition problems below. Then find the answer to the subtraction problem and the integer addition problem to determine the final score.

   45 – 23 = (–23) + 45 =

   17 – 35 = 17 + (–35) =

2. Model each situation using an integer addition or integer subtraction problem.

   Your score on a test was 15, but you can answer extra questions to change your score. You answer 4 more questions and answer them correctly. What is your new score?

   Your score on a test was 12, but you can answer extra questions to change your score. You answer 5 more questions, but answer them incorrectly. What is your new score?

   Your score on a test was 15, but you can answer extra questions to change your score. You answer 5 more questions and answer them correctly. What is your new score?

   Your score on a test was 25, but you can answer extra questions to change your score. You answer 12 more questions and answer them incorrectly. What is your new score?

3. Write a sentence that describes a test that could be modeled by the integer subtraction problem. Then find the answer to the subtraction problem to determine the final score.

   15 – 3 =

   23 – 7 =

   23 – (–7) =

   17– (–35) =

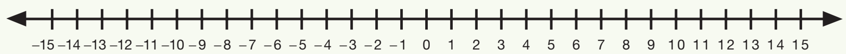
Investigate Problem 1

4. Form a group with another partner team. Compare the differences that you found in Question 3 with others in your group. Then use complete sentences to explain whether describing a test score helped you to understand subtraction of integers.

Problem 2 *Using a Number Line to Subtract Integers*

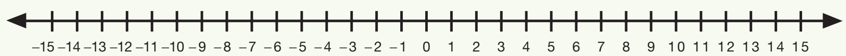
We can use a number line to represent subtraction of integers. Recall from Lesson 7.2 that when you added a negative integer, you moved to the left on the number line. In which direction do you think you should move in order to subtract a positive integer?

A. Start at –4. Then subtract 5. Where are you on the number line?



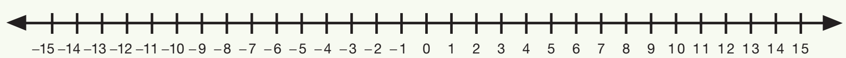
Write and solve an integer subtraction problem that represents this situation.

B. Start at 10. Then subtract 5. Where are you on the number line?



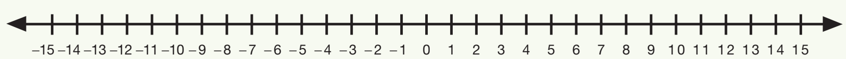
Write and solve an integer subtraction problem that represents this situation.

C. Start at 10. Then subtract –5. Where are you on the number line?



Write and solve an integer subtraction problem that represents this situation.

D. Start at –5. Then subtract –10. Where are you on the number line?



Write and solve an integer subtraction problem that represents this situation.

Investigate Problem 2

1. For each subtraction problem, find the difference. Then write a sentence that describes the movement on the number line that you could use to solve the problem.

   $-18 - 5 =$

   $8 - (-13) =$

   $-240 - 300 =$

   $2450 - (-1500) =$

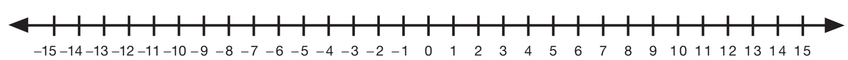
2. **Math Path: Integer Subtraction**

   Complete the rule for using a number line to subtract integers.

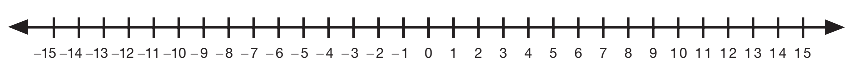
   On a number line, move to the __________ when you subtract a positive integer, and move to the __________ when you subtract a negative integer.

   Use a number line to find each difference.

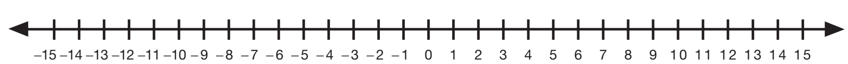
   $-11 - 2 =$



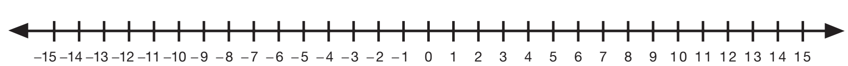
   $5 - (-2) =$



   $-7 - 7 =$



   $-8 - (-8) =$



7.4 Checks and Balances

Multiplying and Dividing Integers

Objectives

In this lesson, you will:

- Multiply integers.
- Divide integers.

Key Terms

- product
- quotient

Problem 1 Checking Account

In a bank account, you can use positive integers to represent deposits, or money that you put into the account. You can use negative integers to represent withdrawals, or money that you take out of the account.

A. You have learned that multiplication is repeated addition. For example, if you deposit \$8 each week for 5 weeks, you can add the number 8 five times, 8 + 8 + 8 + 8 + 8 = 40, or you can multiply 8 by 5 to get 40. You decide to open a checking account. The monthly service charge is \$4, which you can represent as –4. Write the integer multiplication problem represented by the repeated addition in the table. Then find the total amount that has been taken out of your account at the end of each month.

Month	Repeated Addition	Product	Result
1	(–4)	1 × (–4)	–4
2	(–4) + (–4)	2 × (–4)	–8
3	(–4) + (–4) + (–4)		
4	(–4) + (–4) + (–4) + (–4)		
5	(–4) + (–4) + (–4)+ (–4) + (–4)		
6	(–4) + (–4) + (–4)+ (–4) + (–4) + (–4)		

B. Work with your partner to use repeated addition to find each product.

–9 × 6 =

7 × (–11) =

–12 × 7 =

13 × (–5) =

9 × (–1) =

–1 × 6 =

Investigate Problem 1

1. Use a complete sentence to describe what happened in part (B) when you multiplied a number by –1 or multiplied –1 by a number.

2. What is the product of (–1) and 5?

   What is the product of (–1), (–1), and 5?

   What do you think is the product of (–1) and (–1)? Use a complete sentence to explain.

3. Find each product.

3 × (–5) =	(–1) × (–5) =
2 × (–5) =	(–2) × (–5) =
1 × (–5) =	(–3) × (–5) =
0 × (–5) =	

   Use the pattern to determine the sign of the product of two negative numbers. Use a complete sentence in your answer.

4. **Math Path: Product of Negative Integers**

   We have seen that a negative number can be written as the product of –1 and a positive number. For example –6 = –1 × 6. Complete the statement to rewrite each number in the multiplication problem below as the product of –1 and a positive number.

   (–5) × (–4) = (–1) × ☐ × (–1) × ☐

   Now, complete the statement by using the Commutative Property of Multiplication to rewrite the problem.

   (–5) × (–4) = (–1) × (–1) × ☐ × ☐

   You know that (–1) × (–1) = ☐. So, you know that (–4) × (–5) = ☐.

5. Find each product.

7 × (–4) =	(–9) × 4 =
(–8) × (–9) =	(–15) × (–42) =

Problem 2 Account Balances

7

A. You have learned that division is repeated subtraction. For example, if you have $28 in your account and you withdraw $7 each week, you can subtract the number 7 from 28 four times until you have a zero balance: 28 – 7 – 7 – 7 – 7 = 0. You can also divide 28 by 7 to get 4. You are determining the number of 7s there are in 28, which is 4. In a similar way, you can use repeated subtraction to divide integers. For instance, to determine the number of –5s there are in –35, perform repeated subtraction until you get to 0. Complete the table.

Number of –5s	Repeated Subtraction	Result
1	–35 – (–5)	–30
	–35 – (–5) – (–5)	
	–35 – (–5) – (–5) – (–5)	
	–35 – (–5) – (–5) – (–5) – (–5)	
	–35 – (–5) – (–5) – (–5) – (–5) – (–5)	
	–35 – (–5) – (–5) – (–5) – (–5) – (–5) – (–5)	
	–35 – (–5) – (–5) – (–5) – (–5) – (–5) – (–5) – (–5)	

How many –5s there are in –35? Use a complete sentence in your answer.

B. Work with your partner to use repeated subtraction to find each quotient.

–27 ÷ (–9) =

–120 ÷ (–20) =

–45 ÷ (–5) =

C. Another way to find the quotient of two integers is to use what we know about multiplying integers. For example, if we divide 24 by 6, the answer is 4 because 6 × 4 = 24. Use this reasoning to find each quotient by writing a related multiplication problem. Complete each statement.

–27 ÷ (–9) = ☐ because –9 × ☐ = –27.

–27 ÷ 9 = ☐ because 9 × ☐ = –27.

–120 ÷ (–20) = ☐ because –20 × ☐ = –120.

45 ÷ (–5) = ☐ because –5 × ☐ = 45.

Investigate Problem 2

1. Find each quotient by writing and solving a related multiplication problem.

   72 ÷ (–4) =

   –36 ÷ 4 =

   –81 ÷ (–9) =

   –45 ÷ (–15) =

2. **Math Path: Integer Multiplication and Integer Division**

   Form a group with another partner team. In your group, complete the rule for multiplying or dividing two integers.
   Then write an example of the rule.

   The product of two positive integers is a _____________ integer.

   The quotient of two positive integers is a _____________ integer.

   The product of two negative integers is a _____________ integer.

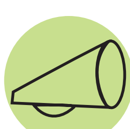
   The quotient of two negative integers is a _____________ integer.

   The product of a positive integer and a negative integer is a _____________ integer.

   The product of a negative integer and a positive integer is a _____________ integer.

   The quotient of a positive integer and a negative integer is a _____________ integer.

   The quotient of a negative integer and a positive integer is a _____________ integer.



3. Use the rules you completed in Question 2 to find each product or quotient.

   –54 ÷ 6 =   144 ÷ 16 =

   55 ÷ (–5) =   –25 × 5 =

   –90 ÷ (–15) =   15 × (–6) =

   22 × 4 =   –12 × (–5) =

7.5 Weight of a Penny

Absolute Value and Additive Inverse

Objectives

In this lesson, you will:

- Write the absolute value of a number.

Key Terms

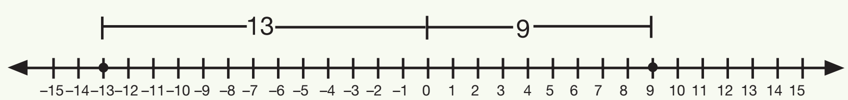
- absolute value
- opposites
- additive inverse

Problem 1 A Science Experiment

The average life span of a U.S. coin is 30 years. In science class, you are weighing pennies from different years to determine whether the mass of a penny changes because of wear and tear. According to the U.S. Mint coin specifications, the mass of a penny should be 2.5 grams, which is 2500 milligrams. Your table shows the differences in the masses of 9 pennies from the specification of 2500 milligrams.

Decade	Mass #1	Mass #2	Mass #3
1970s	–13 milligrams	–14 milligrams	–10 milligrams
1980s	–9 milligrams	–6 milligrams	5 milligrams
1990s	9 milligrams	7 milligrams	–2 milligrams

A. In order to determine which pennies are the furthest from the specification, you can write the *absolute value* of each number in the table. The **absolute value** of a number is the distance between the number and 0 on a number line.



The distance between –13 and 0 is 13, so the absolute value of –13, written as |–13|, is 13. The distance between 9 and 0 is 9, so the absolute value of 9, written as |9|, is 9. Write the absolute value of the number that represents each weight in the table.

Decade	Absolute Value of Mass #1	Absolute Value of Mass #2	Absolute Value of Mass #3
1970s	|–13| = 13	|–14| =	|–10| =
1980s	|–9| =	|–6| =	|5| =
1990s	|9| =	|7| =	|–2| =

B. Which coin is the furthest from specification?

Investigate Problem 1

1. When the expression inside the absolute value symbol is a sum or difference, we find the sum or difference inside the absolute value symbol first. Then we find the absolute value of the result. Work with your partner to find each absolute value.

$|-6 + 5| = |\ \boxed{}\ | =$

$|-7 - (-11)| = |\ \boxed{}\ | =$

$|-7 - 11| = |\ \boxed{}\ | =$

$|5 - 12| = |\ \boxed{}\ | =$

2. There are times when we need to know the distance between any two numbers on the number line. In Lesson 7.2, we used our elevator model to find the sums of integers. Suppose that the elevator traveled from the tenth floor above street level to the third floor below street level. How many floors did the elevator travel? Use a complete sentence in your answer.

How many floors did the elevator travel in going from the fifth floor below street level to the seventh floor above street level? Use a complete sentence in your answer.

Investigate Problem 1

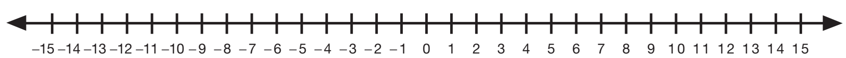
3. In each case in Question 2, we need to find the number of floors traveled, or the distance traveled. Whenever we find a distance traveled, the answer is expressed as a positive number and can be found taking the absolute value of the difference of two numbers. Find the distance between the numbers by writing an absolute value expression.

Distance between 5 and 10 = | ☐ | =

Distance between –5 and 10 = | ☐ | =

Distance between –54 and –23 = | ☐ | =

4. On a number line, when two integers are the same distance from 0 but on opposite sides of 0, the integers are **opposites.** Graph the integers –8, 5, –3, 0, and 12 on the number line. Then graph the opposite of the integer. Zero is its own opposite.



Decide whether the statement below is true or false. Write an example to justify your answer.

Two numbers are opposites if they have the same absolute value but different signs.

5. **Math Path: Additive Inverse**

In earlier lessons, we used the additive identity 0. The **additive inverse** of a number is the number such that the sum of the given number and its additive inverse is 0 (the additive identity). Work with your partner to complete each addition problem.

☐ + 6 = 0   –9 + ☐ = 0   –34 + 34 = ☐

7

7.6 Exploring the Moon

Powers of Ten

Objectives

In this lesson, you will:

- Represent numbers using powers of 10.
- Multiply and divide by powers of 10.

Key Terms

- power
- exponent
- power of ten
- expanded form

In earlier chapters, we worked with powers and exponents, expanded form, and the base-ten decimal system. We can now put these different concepts together in order to explore our universe.

Problem 1 The Earth to the Moon

A. In Lesson 1.6, we saw that a power is used to represent repeated multiplication. For example, 3×3 can be represented by the power 3^2, where 3 is the base of the power and 2 is the exponent. Work with your partner to complete each power of ten.

$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^{\square}$

$1000 = 10^{\square}$

$100 = 10^{\square}$

$10 = 10^{\square}$

$1 = 10^{\square}$

B. In order to have a consistent system, the power of ten that is equal to 1 is defined to be 10^0. In fact, this is true for any base. When any base is raised to the zero power, it is defined to be 1. Complete each statement.

$5^0 = \square$ $\quad 25^0 = \square$ $\quad 50^{\square} = 1$ $\quad 500^{\square} = 1$

C. Recall that we can write a number in expanded form using powers of ten. For example, we can write the distance in miles from Earth to the Moon as:

$$\begin{aligned} 238{,}712 &= (2 \times 100{,}000) + (3 \times 10{,}000) + (8 \times 1000) \\ &\quad + (7 \times 100) + (1 \times 10) + (2 \times 1) \\ &= (2 \times 10^5) + (3 \times 10^4) + (8 \times 10^3) + (7 \times 10^2) \\ &\quad + (1 \times 10^1) + (2 \times 10^0) \end{aligned}$$

Write each number in expanded form using powers of ten.

Diameter of the Moon: 3476 km =

Height of lunar mountains: 25,000 ft =

Moon's surface temperature: 273°F =

Investigate Problem 1

1. Write each decimal in expanded form. Remember that the place values to the right of the decimal point are $\frac{1}{10} = 0.1$, $\frac{1}{100} = 0.01$, $\frac{1}{1000} = 0.001$, etc.

   Ratio of gravity of Moon to gravity of Earth:

   0.167 =

   Ratio of the mass of Moon to mass of Earth:

   0.0123 =

   Number of days for Moon to rotate around Earth:

   27.322 =

2. Form a group with another partner team. Compare your answers with others in your group. Then use complete sentences to explain how you wrote each number in expanded form.

3. In keeping with the system of powers of 10, how do you think we can define powers of ten that are less than 1? Use complete sentence to explain your method.

4. Use the method you explained in Question 3 to complete the following powers.

   $10{,}000 = 10^{\square}$ $\qquad$ $\frac{1}{10} = 10^{\square}$

   $1000 = 10^{\square}$ $\qquad$ $\frac{1}{100} = 10^{\square}$

   $100 = 10^{\square}$ $\qquad$ $\frac{1}{1000} = 10^{\square}$

   $10 = 10^{\square}$ $\qquad$ $\frac{1}{10{,}000} = 10^{\square}$

   $1 = 10^{\square}$

Investigate Problem 1

5. Write each number in expanded form using powers of ten.

23.45 =

345.125 =

56,345.987 =

405,378.34 =

6. Write each number as a power of ten.

10,000,000 = 0.000001 =

1,000,000,000 = 0.0000001 =

7. Math Path: Multiplying and Dividing by Powers of Ten

The table shows the product of a number and a power of 10.

10 × 4.5 = 45	0.1 × 4.5 = 0.45
100 × 4.5 =	0.01 × 4.5 =
1000 × 4.5 =	0.001 × 4.5 =
10,000 × 4.5 =	0.0001 × 4.5 =

Write a sentence explaining how the decimal point moves when you multiply by powers of 10 greater than 1 (a whole number).

Write a sentence explaining how the decimal point moves when you multiply by powers of 10 less than 1 (a decimal).

What is the quotient of 1245 and 100?
What is the quotient of 68 and 0.001?

Complete the statements that give rules for dividing by powers of 10.

When you divide by powers of 10 that are greater than 1, you move the decimal point one place to the _______ for each zero in the power of 10.

When you divide by powers of 10 that are less than 1, you move the decimal point one place to the _______ for each decimal place in the power of 10.

8. Use what you learned in Question 7 to find each product or quotient.

89 × 10 = 209 × 0.1 = 155 × 0.01 =

2.9 × 100 = 5.237 × 1000 = 178 × 0.001 =

251 ÷ 100 = 9.265 ÷ 0.10 = 10,454 ÷ 0.001 =

7

7.7 Expanding Our Perspective

Scientific Notation

7

Objectives

In this lesson, you will:

- Read and write numbers using scientific notation.

Key Terms

- scientific notation
- negative exponent

In the last century, scientists began to unlock the mysteries of the universe by exploring our solar system and galaxy. Through the use of new technologies, scientists also began to explore incredibly small building blocks of matter including cells, molecules, and atoms. Scientists needed to express the very large numbers of our universe and the very small numbers within the subatomic universe as well.

Problem 1 Numbers of the Universe

Scientists began to use a shorthand notation for writing large and small numbers using the powers of ten called **scientific notation.** To use scientific notation, begin by writing the number as a decimal between 1 and 10. Then multiply the decimal by a power of ten.

For example, the distance from the Sun to Earth is approximately 93,000,000 miles. Using scientific notation, we can write this number as $9.3 \times 10{,}000{,}000 = 9.3 \times 10^7$.

In the following table are facts about the universe. The very large numbers are written in standard form or using scientific notation. Complete the table.

Description	Standard Form	Scientific Notation
Distance from Earth to the closest star, Proxima Centauri	39,920,000,000,000 km	
Farthest distance from Earth to Pluto	7,528,000,000 km	
Number of stars in the Milky Way		1.0×10^{14}
Farthest distance to outer ring of Saturn from center of planet	480,000 km	
Farthest distance Voyager 1 was from Earth		1.42×10^{10} km
Distance around the Sun (circumference)	4,400,000 km	

Investigate Problem 1

1. Math Path: Negative Exponents

In Lesson 7.6, we found powers of ten for numbers that were less than 1.

$\frac{1}{10} = 10^{-1}$ $\frac{1}{100} = 10^{-2}$ $\frac{1}{1000} = 10^{-3}$ $\frac{1}{10{,}000} = 10^{-4}$

The exponents on these powers of ten are **negative exponents.** Complete the statements for each power of 10. The first one is done for you.

$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$ $\square = \frac{1}{100} = \frac{1}{10^{\square}} = 10^{\square}$

$\square = \frac{1}{1000} = \frac{1}{10^{\square}} = 10^{\square}$

$\square = \frac{1}{10{,}000} = \frac{1}{10^{\square}} = 10^{\square}$

2. Write each number as a power with a negative exponent. Then find the value of the power.

$\frac{1}{2^3} =$ $\frac{1}{5^2} =$

$\frac{1}{4^2} =$ $\frac{1}{3^3} =$

3. Scientific notation is also used to represent very small numbers. For example, the width of a hydrogen atom is about 0.000000000019253 meters, which can be written in scientific notation as 1.9253×10^{-11} meters.

The table below lists facts about the universe of the atom. The very small numbers are written in standard form or using scientific notation. Complete the table.

Description	Standard Form	Scientific Notation
Width of a human red blood cell	0.000007 m	
Length of microscopic ocean plants called phytoplankton		4.5×10^{-5} m
Width of a grain of sand	0.00012 m	
Diameter of a raindrop	0.003 m	
Thickness of a piece of paper		1.8×10^{-4} m
Diameter of a virus	0.0000001 m	

Looking Back at Chapter 7

Key Terms

integer • p. 199
negative integer • p. 199
positive integer • p. 199
number line • p. 199
profit • p. 201
loss • p. 201
sum • p. 204
integer addition • p. 206
difference • p. 209
integer subtraction • p. 210
product • p. 211
quotient • p. 213
absolute value • p. 215
opposites • p. 217
additive inverse • p. 217
power • p. 219
exponent • p. 219
power of ten • p. 219
expanded form • p. 219
scientific notation • p. 223
negative exponent • p. 224

Summary

Using Integers in Real-Life Situations (p. 199)

Integers allow you to represent quantities in real-life situations in such a way that problems involving these quantities are easier to solve.

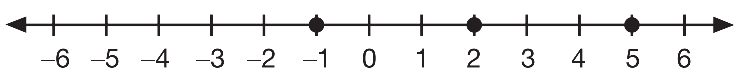
Examples You can represent distances with integers:
loss of 7 yards: –7 yards   5 feet in reverse: – 5 feet

You can represent temperatures with integers:
–35 degrees Fahrenheit   –10 degrees Celsius

Graphing Integers on a Number Line (p. 199)

A number line is a line that extends in both directions forever with one point that is assigned a value of zero and a given length assigned as one unit. To graph an integer, draw a dot on the number line where the integer is represented.

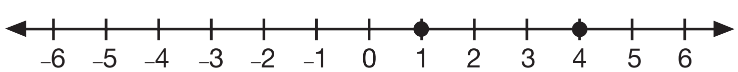
Examples The integers –1, 2, and 5 are graphed below.



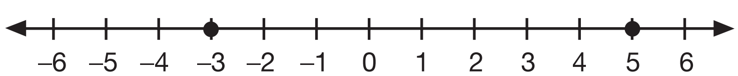
Comparing Integers (p. 199)

To compare integers, graph each integer on a number line. The values of integers increase as you move from left to right.

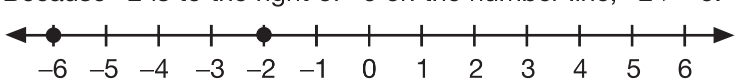
Examples Because 1 is to the left of 4 on the number line, 1 < 4.



Because 5 is to the right of –3 on the number line, 5 > –3.



Because –2 is to the right of –6 on the number line, –2 > –6.



Using a Number Line to Add Integers (p. 205)

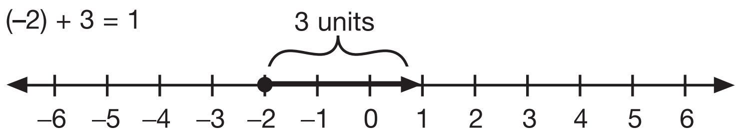
To add two integers using a number line, begin by graphing the first integer in the sum.

If the second number in the sum is *positive*, move to the right on the number line the number of units given by the second number.

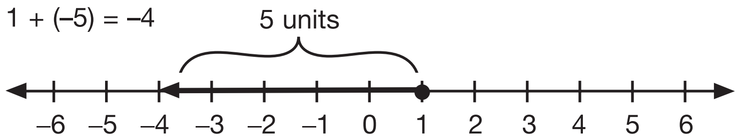
If the second number in the sum is *negative*, move to the left on the number line the number of units given by the second number.

Example

(–2) + 3 = 1



1 + (–5) = –4



Using a Number Line to Subtract Integers (p. 209)

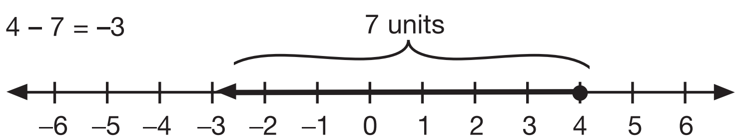
To subtract two integers using a number line, begin by graphing the first integer in the difference.

If the second number in the difference is *positive*, move to the left on the number line the number of units given by the second number.

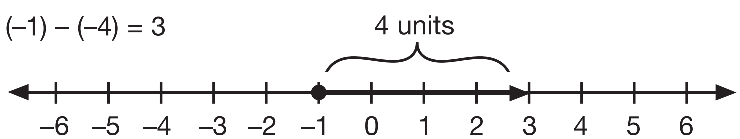
If the second number in the difference is *negative*, move to the right on the number line the number of units given by the second number.

Examples

4 – 7 = –3



(–1) – (–4) = 3



Multiplying and Dividing Integers (p. 214)

Multiplication and division of *positive* integers is the same as with other numbers.

To find a product that involves *negative* integers, first rewrite each negative number as the product of –1 and a positive number. Then find the product using the fact that (–1) × (–1) = 1, when necessary. To divide negative integers, use what you know about multiplying integers.

Examples

(–5) × 6 = (–1) × 5 × 6 = –30

(–8) × (–2) = (–1) × (–1) × 8 × 2 = 16

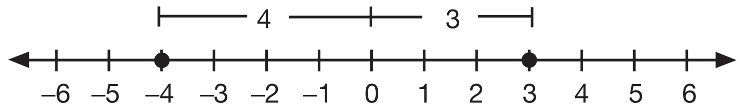
36 ÷ (–6) = –6 because (–6) × (–6) = 36

(–40) ÷ (–4) = 10 because 10 × (–4) = –40

Finding Absolute Value (p. 215)

To find the absolute value of a number, find the distance between the number and 0 on a number line.

Example $|-4| = 4$ $\quad |3| = 3$



Finding the Absolute Value of a Sum or Difference (p. 216)

To find the absolute value of a sum or difference, first find the sum or difference inside the absolute value. Then find the absolute value of the result.

Examples $|-11 + 6| = |-5| = 5$ $\quad |-8 - (-10)| = |2| = 2$

$|-5 - 17| = |-22| = 22$ $\quad |3 - 9| = |-6| = 6$

Using the Additive Inverse (p. 217)

The additive inverse of a number is the number such that the sum of the given number and its additive inverse is 0.

Examples $4 + \square = 0$ $\quad \square + (-16) = 0$ $\quad 17 + (-17) = \square$

$4 + (-4) = 0$ $\quad 16 + (-16) = 0$ $\quad 17 + (-17) = 0$

Multiplying and Dividing by Powers of Ten (p. 221)

When you multiply a number by a power of ten that is *greater than 1*, move the decimal point of the number one place *to the right* for each zero in the power of ten.

When you multiply a number by a power of ten that is *less than 1*, move the decimal point of the number one place *to the left* for each zero in the power of ten.

When you divide a number by a power of ten that is *greater than 1*, move the decimal point of the number one place *to the left* for each zero in the power of ten.

When you divide a number by a power of ten that is *less than 1*, move the decimal point of the number one place *to the right* for each zero in the power of ten.

Examples $100 \times 0.35 = 35$ $\quad 0.01 \times 299 = 2.99$ $\quad 10 \times 6.75 = 67.5$

$4.1 \div 0.001 = 4100$ $\quad 922 \div 100 = 9.22$ $\quad 3 \div 0.1 = 30$

Using Scientific Notation to Write Numbers (p. 224)

To write a number using scientific notation, first write the number as a decimal between 1 and 10. Then multiply the decimal by an appropriate power of ten.

Examples $3{,}600{,}000 = 3.6 \times 10^{6}$ $\quad 45{,}000{,}000{,}000 = 4.5 \times 10^{10}$

$615{,}700 = 6.157 \times 10^{5}$ $\quad 0.0000345 = 3.45 \times 10^{-5}$

$0.00000000042 = 4.2 \times 10^{-10}$ $\quad 0.0099 = 9.9 \times 10^{-3}$