Looking Ahead to Chapter 3

Focus In Chapter 3, you will work with fractions and mixed numbers. You will add, subtract, multiply, and divide fractions and mixed numbers. You will write fractions as mixed numbers and mixed numbers as fractions. You will also learn to work with measurements in customary units and convert measurements.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 3.

Perform the indicated operation.

1. 138 + 587
2. 964 – 26
3. 14×52
4. 39×23
5. $1020 \div 68$
6. $2108 \div 31$

Write a fraction that is equivalent to the given fraction.

7. $\frac{5}{9}$
8. $\frac{17}{21}$
9. $\frac{25}{56}$

Write the fraction in simplest form.

10. $\frac{18}{27}$
11. $\frac{24}{64}$
12. $\frac{36}{108}$

Read the problem scenario below.

You ride a bicycle with tires that are 26 inches in diameter. You pedal at a rate of 50 revolutions per minute. You can travel about 340 feet in one minute.

13. Which units are used to measure distance?
14. Which units are used to measure time?

Key Terms

like fractions ● p. 75
unlike fractions ● p. 75
least common denominator ● p. 78
improper fractions ● p. 82
mixed number ● p. 82
U.S. customary system ● p. 85
metric system ● p. 85
remainder ● p. 89
multiplicative identity ● p. 91
multiplicative inverse ● p. 91
reciprocal ● p. 91
customary units of measure ● p. 101
length ● p. 101
inch ● p. 101
foot ● p. 101
yard ● p. 101
mile ● p. 101
capacity ● p. 101
fluid ounce ● p. 101
cup ● p. 101
pint ● p. 101
quart ● p. 101
gallon ● p. 101
weight ● p. 101
ounce ● p. 101
pound ● p. 101
ton ● p. 101

3

Operations with Fractions and Mixed Numbers

People in the U.S. eat an average of 4.5 pounds of strawberries each year. In Lesson 3.5, you will divide quarts of berries among people in order to learn about fraction division.

3.1

Who Gets What?

Adding and Subtracting Fractions with Like Denominators

Objectives

In this lesson, you will:

- Add and subtract like fractions.

Key Terms

- like fractions
- unlike fractions

Problem 1 Understanding Inheritance

After humankind began dividing wholes into parts, they had two problems:

How much do two or more people have altogether?

How much more does one person have than another?

For example, when a person dies, his or her possessions are divided among his or her children. The part that each child receives is called an inheritance.

A. In a family with two children, the oldest child receives two fifths of an inheritance and the youngest child receives one fifth. What fraction of the total inheritance do they receive altogether? Use your fraction strips to model the problem and find the answer.

B. Suppose that the oldest child receives three eighths of an inheritance and the youngest child receives two eighths. What fraction of the total inheritance do they receive altogether?

C. Suppose that four children in a family each receive $\frac{2}{9}$ of an inheritance. What fraction of the total inheritance do they receive altogether?

D. In parts (A), (B), and (C), you were adding fractions. Write your answer to each fraction addition problem below.

$\frac{2}{5} + \frac{1}{5} =$ $\quad\quad$ $\frac{3}{8} + \frac{2}{8} =$ $\quad\quad$ $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} =$

E. Study each problem and answer. Then write complete sentences that explain how you can find the answer to a fraction addition problem without using fraction strips.

F. In each problem, what do you notice about the denominators of the addends and the denominator of the sum? Explain what you notice using complete sentences.

Take Note

Remember, when you add two numbers to produce another number, each number that you add is an *addend*.

Investigate Problem 1

1. Suppose that a child receives $\frac{4}{5}$ of an inheritance and a nephew receives $\frac{1}{5}$ of the inheritance. Use fraction subtraction to find the difference in the fractions of the inheritance that the child and the nephew receive. You may want to use your fraction strips to model the problem.

2. In California, if a person who has a spouse and four children dies with no will, then the spouse receives $\frac{4}{12}$ of the inheritance and each child receives $\frac{2}{12}$. Find the difference in the fractions of the inheritance that the spouse receives and one child receives. Simplify your answer, if possible.

3. In Massachusetts, if a person who has a spouse and three children dies with no will, then the spouse receives $\frac{3}{6}$ of the inheritance and each child receives $\frac{1}{6}$. Find the difference in the fraction of the inheritance that the spouse receives and one child receives. Simplify your answer, if possible.

4. In Questions 1, 2, and 3, you were subtracting fractions. For each question, write a fraction subtraction problem. Then find each difference.

5. Study each problem and answer. Then write complete sentences that explain how you can find the answer to a fraction subtraction problem without using fraction strips.

6. In each problem, what do you notice about the denominators of the fractions? Explain what you notice using complete sentences.

Investigate Problem 1

7. Math Path: Like and Unlike Fractions

Fractions that have the same denominator are called **like fractions.** Fractions that have different denominators are called **unlike fractions.** Which kind of fractions have we been adding and subtracting so far in this lesson? Work with your partner to write a rule for adding like fractions. Then write a rule for subtracting like fractions.

Rule for adding like fractions:

Rule for subtracting like fractions:

8. Use the rules that you wrote in Question 7 to find each sum and/or difference. Simplify your answer, if possible.

$\frac{2}{5} + \frac{2}{5} =$ $\qquad$ $\frac{2}{7} + \frac{3}{7} =$ $\qquad$ $\frac{1}{8} + \frac{1}{8} =$

$\frac{6}{7} - \frac{3}{7} =$ $\qquad$ $\frac{1}{10} + \frac{3}{10} =$ $\qquad$ $\frac{8}{9} - \frac{2}{9} =$

$\frac{2}{7} + \frac{2}{7} + \frac{2}{7} =$ $\qquad$ $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} =$

$\frac{1}{10} + \frac{3}{10} + \frac{3}{10} + \frac{1}{10} =$ $\qquad$ $\frac{5}{8} + \frac{1}{8} + \frac{1}{8} - \frac{3}{8} =$

$\frac{1}{12} + \frac{1}{12} + \frac{5}{12} =$ $\qquad$ $\frac{1}{16} + \frac{7}{16} - \frac{5}{16} =$

Take Note

Remember that the order of operations tells you to add and subtract real numbers from left to right.

3.2 Old-Fashioned Goodies

Adding and Subtracting Fractions with Unlike Denominators

Objectives

In this lesson, you will:

- Add and subtract unlike fractions.

Key Terms

- least common denominator

Problem 1 Candy Store

Your aunt is opening a store that sells old-fashioned goodies. She wants to offer molasses candy, hardtack candy, and popcorn balls. She has a recipe for popcorn balls that uses $\frac{1}{3}$ cup of corn syrup and a recipe that uses $\frac{1}{2}$ cup of corn syrup. If she wants to make one batch of each recipe, how much corn syrup does she need altogether?

A. Work with your partner and use your fraction strips to model the problem. Can you find an exact answer using just your $\frac{1}{2}$ and $\frac{1}{3}$ fraction strips? Use complete sentences to explain your answer.

B. Try using other fraction strips to find the answer. Were you able to find an exact answer? Use complete sentences to explain why or why not.

C. Form a group with another partner team. Take turns sharing how you solved the problem.

D. Which fraction strip helped you solve the problem? Is there only one fraction strip that will work? Use complete sentences to explain.

E. Use the fraction strip from part (D) to find an equivalent fraction for $\frac{1}{2}$. Then use this fraction strip to find an equivalent fraction for $\frac{1}{3}$. Now use your rule from Lesson 3.1 to add these two like fractions. Is your answer the same as your answer from Part (B)?

Investigate Problem 1

Work in your group and use your fraction strips to solve each problem.

1. Hardtack candy is an old-fashioned type of candy that uses flavoring oils like peppermint or cinnamon. Suppose you have two recipes for a batch of hardtack candy. One recipe calls for a dram (or $\frac{1}{4}$ tablespoon) of flavoring oil and another recipe uses a teaspoon (or $\frac{1}{3}$ tablespoon) of flavoring oil. How much total flavoring oil do you need to make one batch of both recipes? Use a complete sentence to write your answer.

2. A recipe for maple candy uses $\frac{3}{8}$ cup of brown sugar and a recipe for molasses candy uses $\frac{1}{2}$ cup of brown sugar. How much total brown sugar do you need to make one batch of both kinds of candy? Use a complete sentence to write your answer.

3. In your group, discuss how you solved these problems. For each problem, identify which fraction strips you used and explain your choices.

4. In your group, write a rule for adding unlike fractions.

Rule for adding unlike fractions:

5. Math Path: Least Common Denominator

You have seen that adding unlike fractions is different from adding like fractions. With like fractions, you can add the numerators to find your answer because the wholes are divided into the same number of parts. When you add unlike fractions, you cannot just add the numerators because the wholes are divided into different numbers of parts. So, you must first write the fractions as equivalent fractions that have the same number of parts, or denominator. It is most efficient to use the **least common denominator** (LCD), although any common denominator will work. Why is it more efficient to use the LCD? Use complete sentences to explain.

Investigate Problem 1

6. Often it is easier to write fraction addition problems vertically. Let's do the fraction addition problems on the previous pages using this method. Start by finding the LCD. Why? Next, write equivalent fractions that have the LCD as their denominators. Finally, add the resulting fractions.

$$\text{LCD} = \boxed{\bullet\bullet}$$
$$\begin{array}{rl} & \frac{1}{2} = \frac{1\times\boxed{\bullet}}{2\times\square} = \frac{\square}{\square} \\ + & \frac{1}{3} = \frac{1\times\square}{3\times\square} = \frac{\square}{\square} \\ \hline & \qquad\qquad\quad = \frac{\square}{\square} \end{array}$$

$$\text{LCD} = \boxed{\bullet\bullet}$$
$$\begin{array}{rl} & \frac{1}{3} = \frac{1\times\boxed{\bullet}}{3\times\square} = \frac{\square}{\square} \\ + & \frac{1}{4} = \frac{1\times\square}{4\times\square} = \frac{\square}{\square} \\ \hline & \qquad\qquad\quad = \frac{\square}{\square} \end{array}$$

$$\text{LCD} = \boxed{\bullet\bullet}$$
$$\begin{array}{rl} & \frac{3}{8} = \frac{3\times\boxed{\bullet}}{8\times\square} = \frac{\square}{\square} \\ + & \frac{1}{2} = \frac{1\times\square}{2\times\square} = \frac{\square}{\square} \\ \hline & \qquad\qquad\quad = \frac{\square}{\square} \end{array}$$

7. When we subtract unlike fractions, we follow a similar procedure. If a child got $\frac{1}{2}$ of an inheritance but gave away $\frac{1}{3}$ of it, how much would he have left? Use your fraction strips to model this problem. Do we have the same type of dilemma that we had when we added $\frac{1}{2}$ and $\frac{1}{3}$? Use an additional fraction strip to solve the problem. Which fraction strip did you use? Why? Use the space below to write the steps you need to follow to subtract unlike fractions.

8. Suppose that Erica had $\frac{7}{8}$ of a pizza and gave your little brother Manuel $\frac{1}{4}$. How much does Erica have left? Do this problem numerically below and write an explanation of what you are doing and why so that Erica can explain it to Manuel.

$$\text{LCD} = \boxed{\bullet\bullet}$$
$$\begin{array}{rl} & \frac{7}{8} = \frac{7\times\boxed{\bullet}}{8\times\square} = \frac{\square}{\square} \\ - & \frac{1}{4} = \frac{1\times\square}{4\times\square} = \frac{\square}{\square} \\ \hline & \qquad\qquad\quad = \frac{\square}{\square} \end{array}$$

Investigate Problem 1

9. Find each difference numerically. Simplify your answer, if possible.

$\frac{5}{6} - \frac{1}{3} =$

$$\text{LCD} = \boxed{\bullet\bullet}$$

$$\begin{array}{rcl} \frac{5}{6} = \frac{5\times\boxed{\bullet}}{6\times\boxed{}} & = & \frac{\boxed{}}{\boxed{}} \\ -\frac{1}{3} = \frac{1\times\boxed{}}{3\times\boxed{}} & = & \frac{\boxed{}}{\boxed{}} \\ \hline & = & \frac{\boxed{}}{\boxed{}} \end{array}$$

$\frac{19}{24} - \frac{1}{2} =$

$$\text{LCD} = \boxed{\bullet\bullet}$$

$$\begin{array}{rcl} \frac{19}{24} = \frac{19\times\boxed{}}{24\times\boxed{}} & = & \frac{\boxed{}}{\boxed{}} \\ -\frac{1}{2} = \frac{1\times\boxed{}}{2\times\boxed{}} & = & \frac{\boxed{}}{\boxed{}} \\ \hline & = & \frac{\boxed{}}{\boxed{}} \end{array}$$

10. Find each sum or difference.

$\frac{11}{12} - \frac{2}{3} =$

$\frac{1}{12} + \frac{1}{4} =$

$\frac{11}{18} + \frac{1}{3} =$

$\frac{13}{15} - \frac{2}{3} =$

$\frac{1}{6} + \frac{2}{5} =$

$\frac{3}{10} + \frac{2}{5} =$

$\frac{5}{16} + \frac{1}{4} =$

$\frac{17}{18} - \frac{2}{3} =$

$\frac{3}{4} + \frac{1}{8} + \frac{1}{8} =$

3.3

Fun and Games

Improper Fractions and Mixed Numbers

Objectives

In this lesson, you will:

- Write improper fractions as mixed numbers.
- Write mixed numbers as improper fractions.

Key Terms

- improper fraction
- mixed number

Problem 1 Making a Spinner

You and your friend have created your own board game. Now you need to make a circular spinner for the game. You want the spinner to have two colors, red and blue. You divide the spinner into 5 equal parts and paint $\frac{3}{5}$ of the spinner red and $\frac{2}{5}$ of the spinner blue.

A. In the game, Player 1 moves a red game piece 1 space if the spinner lands on red. Otherwise, Player 1 does not move the game piece. In a similar way, Player 2 moves a blue game piece 1 space if the spinner lands on blue. Otherwise, Player 2 does not move the game piece. Is the spinner fair? Use complete sentences to explain why or why not.

B. Write the fraction addition problem that represents the total part of the spinner that is painted.

C. What do you know about a fraction whose numerator and denominator are equal? Explain your answer using complete sentences.

D. What if you wanted to paint the spinner so that $\frac{3}{5}$ of the spinner was red and $\frac{3}{5}$ of the spinner was blue? Is this possible? Use complete sentences to explain.

Investigate Problem 1

1. Write the fraction addition problem that represents the situation in Part (D) of Problem 1 and find the answer.

2. Is your answer to Question 1 less than, equal to, or greater than 1? How do you know? Use complete sentences to explain.

3. What can you conclude about a fraction whose numerator is greater than its denominator? Write your answer using a complete sentence.

4. **Math Path: Improper Fractions**

A fraction whose numerator is greater than its denominator is called an **improper fraction.** Usually we do not write an answer as an improper fraction. Instead, we simplify the answer by writing the improper fraction as a **mixed number.** A mixed number is a number that is the sum of a whole number and a fraction. For instance, you can write your answer to Question 1 as the sum of a whole number and a fraction. Because you know that $\frac{5}{5}$ is equal to one whole, you can write $\frac{6}{5}$ as $\frac{5}{5} + \frac{1}{5}$, or 1 whole and $\frac{1}{5}$ which is the mixed number $1\frac{1}{5}$.

Write each improper fraction as a mixed number.

$\frac{7}{4} =$

$\frac{9}{8} =$

$\frac{12}{5} =$

$\frac{9}{4} =$

$\frac{16}{9} =$

$\frac{11}{4} =$

$\frac{14}{3} =$

$\frac{25}{6} =$

Take Note

Remember when you learned long division? When you divided and got a remainder, you were told to write the remainder over the divisor. For example, when you divided 27 by 5, you were really just writing the improper fraction $\frac{27}{5}$ as $5\frac{2}{5}$.

$$\begin{array}{r} 5\frac{2}{5} \\ 5\overline{)\,27} \\ \underline{25} \\ 2 \end{array}$$

5. Find the sum. Simplify your answer, if possible.

$\frac{3}{4} + \frac{3}{4} =$

$\frac{7}{8} + \frac{5}{8} =$

$\frac{5}{6} + \frac{1}{6} =$

$\frac{1}{4} + \frac{5}{6} =$

$\frac{3}{4} + \frac{1}{3} =$

$\frac{5}{8} + \frac{2}{3} =$

Problem 2 Making Jump Ropes

You are making jump ropes for your cousins. You need $1\frac{1}{4}$ yards of rope for your older cousin Teesha's jump rope and $\frac{3}{4}$ yard of rope for your younger cousin Samantha's jump rope. How much more rope do you need for Teesha's jump rope?

A. Write the subtraction problem that represents this situation.

B. To subtract $\frac{3}{4}$ from $1\frac{1}{4}$, you cannot subtract the numerator 3 from the numerator 1, so you need to regroup $1\frac{1}{4}$ as $\frac{4}{4}$ and $\frac{1}{4}$. Fill in the blanks to regroup, then subtract.

$$1\frac{1}{4}-\frac{3}{4}=\left(\frac{4}{4}+\frac{1}{4}\right)-\frac{3}{4}=\frac{\square}{4}-\frac{3}{4}=\frac{\square}{4}=\frac{\square}{\square}$$

Take Note

Regrouping a mixed number to subtract is similar to regrouping to subtract whole numbers. For instance, to subtract 19 from 23, you need to regroup 23 as 1 ten and 13 ones so that you can subtract 9 from 13.

$$\begin{array}{r} \overset{1}{\cancel{2}}\,\overset{13}{\cancel{3}} \\ -1\;9 \\ \hline 4 \end{array}$$

Investigate Problem 2

1. To write a mixed number as an improper fraction, we need to write the whole number as an equivalent number of fractional parts and then add the like fractions. Fill in the blanks to write $2\frac{1}{5}$ as an improper fraction.

$$2\frac{1}{5}=2+\frac{1}{5}=\frac{\square}{5}+\frac{1}{5}=\frac{\square}{5}$$

2. Write each mixed number as an improper fraction.

$3\frac{1}{4}=$ $\qquad$ $1\frac{3}{8}=$ $\qquad$ $2\frac{1}{7}=$

$1\frac{5}{6}=$ $\qquad$ $5\frac{1}{3}=$ $\qquad$ $3\frac{1}{9}=$

3. Find each difference.

$1\frac{3}{8}-\frac{5}{8}=$ $\qquad$ $1\frac{3}{5}-\frac{4}{5}=$ $\qquad$ $1\frac{1}{8}-\frac{3}{4}=$

$1\frac{1}{3}-\frac{5}{6}=$ $\qquad$ $1\frac{3}{7}-\frac{5}{7}=$ $\qquad$ $1\frac{2}{9}-\frac{2}{3}=$

3.4 Parts of Parts

Multiplying Fractions

Objectives

In this lesson, you will:

- Multiply fractions.

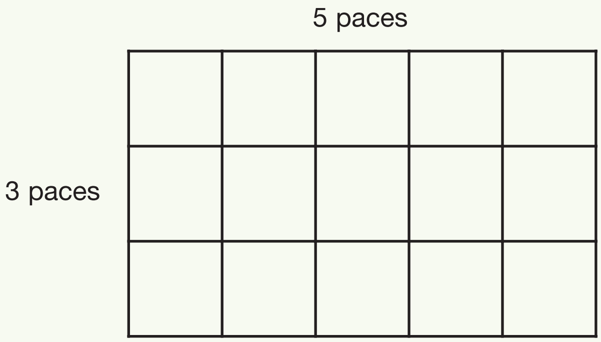
Key Terms

- U.S. customary system
- metric system

When humankind began to group together in societies, the concept of ownership and inheritance became very important. Eventually as the population increased, there was a need to extend this ownership to plots of land.

Problem 1 The First Gardens

A. A rectangle was likely one of the first shapes that humans used to designate plots of land. They measured the plots by walking and measuring the distance with the number of steps (or paces) they had to take. You probably remember that the area of a rectangle can be found by multiplying its length by its width. In the plot below, the length is 5 paces and the width is 3 paces. What is the area of the plot?



B. What are the units for the area of the plot? Use complete sentences to explain your choice.

C. The problem that eventually occurred was that different people have different paces. Why would this be a problem? Write your answer using a complete sentence.

D. Eventually people developed whole systems of measurement to ensure consistency. The two measurement systems that are used today are the **United States customary system** and the **metric system.** Write as many units as you know in the U.S. customary system and in the metric system.

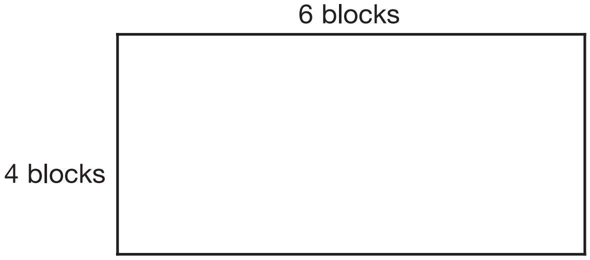
U.S. customary system:

Metric system:

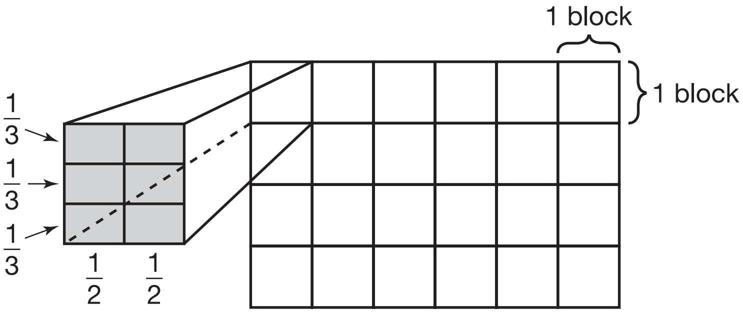
You will learn more about each of these systems in this chapter and in Chapter 4.

Investigate Problem 1

1. A group of students at your school decides to turn a plot of land in the city into a community garden. Find the area of the plot of land below. Be sure to include the units in your answer.



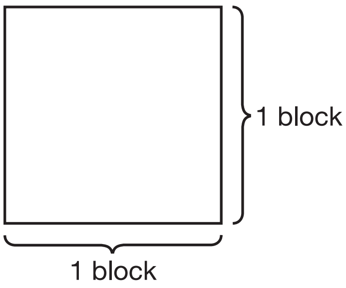
2. As part of the group, your job is to divide each square block into garden plots. How do you find the area of a part of a block? Suppose you need to find the area of a plot that has a length of $\frac{1}{2}$ of a block and a width of $\frac{1}{3}$ of a block. Use the diagram below to determine the number of parts that one square block can be divided into in this way.



3. What does each of these parts represent? What is the area of a part? Use complete sentences to answer each question.

4. Represent this problem as a fraction multiplication problem. Then find the product.

5. What is the area of a plot that is $\frac{2}{3}$ block long and $\frac{1}{2}$ block wide? Use the square to draw a diagram that represents this problem.

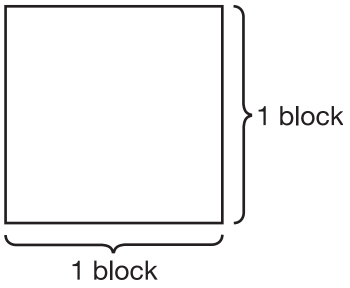


Investigate Problem 1

6. Write a fraction multiplication problem that represents the situation in Question 5.

7. What is the area of a plot that is $\frac{2}{3}$ mile wide and $\frac{3}{4}$ mile long?

Use the diagram below. Explain how you found your answer. Then write the fraction multiplication problem that represents this situation. Simplify your answer, if possible.



8. Form a group with another partner team. Review the fraction multiplication problems that you wrote. Then work with your group to determine a procedure for finding the answers without using a diagram. Explain your procedure using complete sentences.

9. Each group should take turns sharing their procedure with the entire class. Does everyone agree with your group's procedure?

10. Use your procedure to multiply each fraction. Write each answer in simplest form.

$\frac{2}{3} \times \frac{2}{5} =$

$\frac{3}{8} \times \frac{5}{7} =$

$\frac{3}{4} \times \frac{5}{6} =$

$\frac{4}{5} \times \frac{5}{8} =$

Investigate Problem 1

11. Examine the methods used below to multiply and simplify $\frac{3}{8}\times\frac{4}{9}$.

$$\frac{3}{8}\times\frac{4}{9}=\frac{12}{72}=\frac{12\div12}{72\div12}=\frac{1}{6}$$

$$\frac{3}{8}\times\frac{4}{9}=\frac{12}{72}=\frac{{}^{1}\cancel{12}\times1}{{}_{1}\cancel{12}\times6}=\frac{1}{6}$$

$$\frac{3}{8}\times\frac{4}{9}=\frac{{}^{1}\cancel{3}\times{}^{1}\cancel{2}\times{}^{1}\cancel{2}}{{}_{1}\cancel{2}\times{}_{1}\cancel{2}\times2\times{}_{1}\cancel{3}\times3}=\frac{1}{6}$$

$$\frac{{}^{1}\cancel{3}}{{}_{2}\cancel{8}}\times\frac{\cancel{4}^{1}}{\cancel{9}_{3}}=\frac{1}{6}$$

$$\frac{3}{8}\times\frac{4}{9}=\frac{{}^{1}\cancel{12}}{{}_{6}\cancel{72}}=\frac{1}{6}$$

$$\frac{3}{8}\times\frac{4}{9}=\frac{{}^{4}\cancel{12}}{{}_{24}\cancel{72}}=\frac{{}^{1}\cancel{4}}{{}_{6}\cancel{24}}=\frac{1}{6}$$

Discuss each of the methods with your group. Do all methods produce the same answer?

How are the methods alike? How are the methods different? Use complete sentences to answer each question.

Are all of the methods correct? Use complete sentences to explain your answer.

What conclusions can you make about the procedure for multiplying fractions?

3.5 Parts in a Part

Dividing Fractions

Objectives

In this lesson, you will:

- Divide fractions.

Key Terms

- remainder
- multiplicative identity
- multiplicative inverse
- reciprocal

Take Note

Recall that the **remainder** is the whole number left over in a division problem if the divisor does not divide the dividend evenly. For example, when 17 is divided by 3, the remainder is 2.

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{15} \\ 2 \end{array}$$

When people began to divide plots of land to give to their children, they often gave the first born son more than the rest of their sons. A father wants to leave his plot of land to his three sons so that the first born will inherit three-fourths of the plot rather than giving each son the same amount. The second son says that this is unfair because the first born will get 3 times as much than if it was divided evenly. The third son says no, it will be 2 times as much. Which son is correct? How many times more will the first son receive than his brothers than if the plot was divided evenly? In other words, how many one-thirds are in three-fourths? We can find out by solving this division problem: $\frac{3}{4} \div \frac{1}{3}$.

Problem 1 Dividing Berries

To find the answer to the brother's problem, let's first review what we know about division.

A. Remember that one way to think about division is to think of it as repeated subtraction. For example, to solve the division problem 12 ÷ 3, ask yourself, "How many times can I take a group of 3 quarts of berries away from a group of 12 quarts of berries?" Draw a diagram to explain your answer.

B. Another way to think about the division problem 12 ÷ 3 is to think of dividing 12 quarts of berries evenly among 3 people. This strategy works pretty well when the number you are dividing by is a factor of the dividend, but what happens when we need to divide 13 quarts of berries evenly among 3 people? How much does each person receive? Explain to your partner how you found your answer.

C. You may remember that when you first learned about division, you divided 13 by 4 and wrote the answer as 3 with a remainder of 1. Later you were probably told to express the amount left over as a fraction by writing the remainder over the divisor. Does it make sense to you now why you wrote the amount left over as a fraction? Explain this to your partner, and listen to his or her explanation. Do the two of you agree? Explain why or why not using complete sentences.

Investigate Problem 1

1. Before we solve the fraction division problem $\frac{3}{4} \div \frac{1}{3}$, consider $\frac{3}{4} \div \frac{1}{2}$. Another way to think of this problem is to ask, "How many $\frac{1}{2}$s are there in $\frac{3}{4}$? Use your fraction strips and your knowledge of subtracting fractions to determine how many times you can subtract $\frac{1}{2}$ from $\frac{3}{4}$. How much will be left over? This is the remainder. Use complete sentences in your answers.

2. Consider $\frac{2}{4} \div \frac{1}{2}$. Use your fractions strips to determine the number of $\frac{1}{2}$s there are there in $\frac{2}{4}$. How much is left over? Use a complete sentence to answer the question.

3. In both Questions 1 and 2, the whole number parts in the answer are the same, but with $\frac{3}{4} \div \frac{1}{2}$ we have a remainder. If we were working with whole numbers, we would simply write the amount left over as a fraction with the remainder as the numerator and the divisor as the denominator. In Question 1, the remainder is a fraction and the divisor is also a fraction. Write the amount left over as a fraction divided by a fraction.

Amount left over: $$\cfrac{\cfrac{\square}{\square} \leftarrow \text{Remainder}}{\cfrac{\square}{\square} \leftarrow \text{Denominator}}$$

4. According to the definition of a fraction, the question we want to answer is "What part of the denominator is represented by the numerator?" Write both the numerator and denominator as like fractions. Can you answer the question now? Work with your partner to determine the final answer to the following fraction division problem. Then share how you found your answer with another partner team.

$\frac{3}{4} \div \frac{1}{2} =$

Investigate Problem 1

5. Now let's solve our original problem, $\frac{3}{4} \div \frac{1}{3}$. How many $\frac{1}{3}$s are there in $\frac{3}{4}$? What amount is left over? What part of $\frac{1}{3}$ is this? Use complete sentences to explain how you know.

6. **Math Path: Dividing Fractions**

Using either fraction strips or like denominators to divide fractions will become difficult as we work with fractions with larger denominators. There is a more efficient and easier method for dividing fractions. This method works because of some special properties that we discovered earlier:

Division by 1: Whenever we divide any number by 1, the answer is always the number itself.

$a \div 1 = a$

Multiplicative identity: The product of any number and 1 is the number. So, the **multiplicative identity** is the number 1.

$a \times 1 = a$

Fractions equal to 1: Any fraction whose nonzero numerator and nonzero denominator are the same is equal to 1.

$\frac{a}{a} = 1$

Multiplicative inverse: The product of any nonzero number and its multiplicative inverse is 1. The **multiplicative inverse** of a number is also known as the **reciprocal** of the number.

$\frac{a}{b} \times \frac{b}{a} = 1$

Examine the following fraction division problem.

$$\frac{3}{4} \div \frac{1}{2} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{1}{2}} \times \frac{\frac{2}{1}}{\frac{2}{1}} = \frac{\frac{3 \times 2}{4 \times 1}}{\frac{1 \times 2}{2 \times 1}} = \frac{\frac{6}{4}}{1} = \frac{{}^{3}\cancel{6}}{{}_{2}\cancel{4}} = \frac{3}{2} = 1\frac{1}{2}$$

Fractions equal to 1: $\frac{\frac{2}{1}}{\frac{2}{1}}$ Multiplicative identity: $\frac{\frac{3}{4}}{\frac{1}{2}} \times \frac{\frac{2}{1}}{\frac{2}{1}}$

Multiplicative inverse: $\frac{1 \times 2}{2 \times 1}$ Division by 1: $\frac{\frac{6}{4}}{1}$

Circle each property listed above in the fraction division problem.

Investigate Problem 1

7. Compare the problem in Question 6 with the following problem.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{{}^{3}\cancel{6}}{{}_{2}\cancel{4}} = \frac{3}{2} = 1\frac{1}{2}$$

Is the method used above the same as the method used in Question 6? How do you know? Which steps have been removed? Use complete sentences to write your answers.

8. Use the method demonstrated in Question 7 to solve the fraction division problem $\frac{3}{4} \div \frac{1}{3}$.

$\frac{3}{4} \div \frac{1}{3} =$

9. Use the method in Question 7 to find each quotient.

$\frac{3}{8} \div \frac{3}{4} =$

$\frac{5}{6} \div \frac{2}{3} =$

$\frac{7}{8} \div \frac{3}{4} =$

$\frac{11}{12} \div \frac{1}{3} =$

$\frac{9}{10} \div \frac{2}{5} =$

3.6

All That Glitters

Adding and Subtracting Mixed Numbers

Objectives

In this lesson, you will:

- Add and subtract mixed numbers.

Key Terms

- mixed number

In Lesson 3.3, we said that a *mixed number* has a whole number part and a fractional part. Because we know how to add, subtract, multiply, and divide fractions, we can now solve problems involving operations with mixed numbers.

Problem 1 Gold Ingots

Suppose that you are a treasure hunter searching a Spanish shipwreck from the 1600s and you find gold bars, called ingots. The treasure consists of whole gold ingots, one-half ingots, and one-third ingots. If you find $3\frac{1}{3}$ gold ingots on the first day and $1\frac{1}{3}$ gold ingots on the second day, what is the total number of ingots that you have?

A. Represent this problem by drawing a diagram below of the amount you found on the first day and the second day.

B. How many full bars do you have? What fractional part of a bar do you have? Use a complete sentence to write your answer.

C. Write your total as a single mixed number. Then use complete sentences to explain how you can find this total without drawing a picture.

Investigate Problem 1

1. Use the method you described in part (C) to find each sum.

$2\frac{1}{3} + 3\frac{1}{3} =$ $1\frac{2}{5} + 4\frac{2}{5} =$ $5\frac{3}{8} + 4\frac{1}{4} =$

$3\frac{1}{3} + 3\frac{1}{6} =$ $4\frac{1}{3} + 3\frac{2}{3} =$ $5\frac{4}{7} + 2\frac{3}{7} =$

2. If you find $2\frac{3}{5}$ gold ingots and then find an additional $3\frac{3}{5}$ ingots, how many ingots do you have altogether? Draw a diagram that represents the problem. Use your diagram to answer Question 3.

3. How many full bars do you have? What fractional part of a bar do you have? Write your answer using a complete sentence.

4. Write your total as a single mixed number. Then use complete sentences to explain how you can get this answer without drawing a picture.

5. Use the method you described in Question 4 to find each sum.

$3\frac{2}{3} + 2\frac{2}{3} =$ $2\frac{4}{5} + 2\frac{4}{5} =$ $3\frac{7}{8} + 5\frac{1}{4} =$

$5\frac{2}{3} + 6\frac{5}{6} =$ $9\frac{2}{3} + 3\frac{5}{9} =$ $7\frac{5}{5} + \frac{4}{9} =$

Investigate Problem 1

6. What if you found $3\frac{2}{3}$ ingots and give your friend $1\frac{1}{3}$ of them? Draw a diagram that represents the problem. Use your diagram to answer Question 7.

7. How many full bars do you have? What fractional part of a bar do you have? Write your answer using a complete sentence.

8. Write your total as a single mixed number. Then use complete sentences to explain how you can get this answer without drawing a picture.

9. Use the method you described in Question 8 to find each difference.

$3\frac{2}{3} - 1\frac{1}{3} =$ $\quad$ $5\frac{4}{5} - 2\frac{3}{5} =$ $\quad$ $5\frac{7}{8} - 2\frac{3}{8} =$

$6\frac{2}{3} - 5\frac{1}{6} =$ $\quad$ $3\frac{2}{5} - 3\frac{3}{10} =$ $\quad$ $7\frac{4}{5} - \frac{2}{3} =$

3

Investigate Problem 1

10. What if you found $3\frac{1}{3}$ ingots and give your friend $1\frac{2}{3}$ of them?

Draw a diagram that represents the problem. Use your diagram to answer Question 11.

11. How many full bars do you have? What fractional part of a bar do you have? Write your answer using a complete sentence.

12. Write your total as a single mixed number. What additional step do you need to include when performing the subtraction? Use complete sentences to explain how you can get this answer without drawing a picture.

13. Use the method you described in Question 12 to find each difference.

$4\frac{1}{3} - 2\frac{2}{3} =$

$4\frac{1}{8} - \frac{3}{8} =$

$9\frac{1}{5} - 2\frac{9}{10} =$

$6\frac{7}{12} - 3\frac{7}{8} =$

$4\frac{2}{5} - 3\frac{3}{5} =$

$4\frac{1}{3} - 1\frac{5}{6} =$

$8\frac{1}{5} - 4\frac{2}{3} =$

$5\frac{1}{6} - 1\frac{5}{9} =$

3.7 Project Display

Multiplying and Dividing Mixed Numbers

Objectives

In this lesson, you will:

- Multiply and divide mixed numbers.

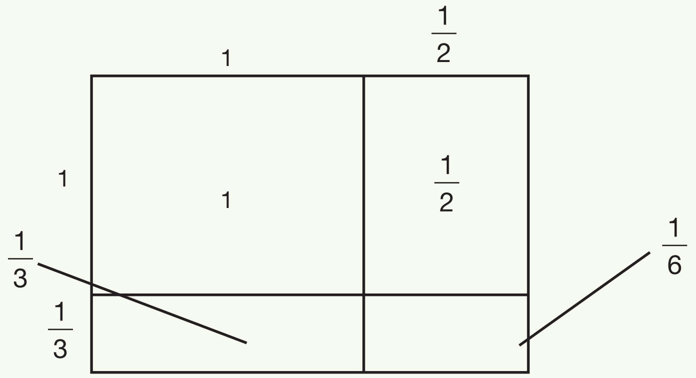
Key Terms

- reciprocal

You may remember that when we multiplied two fractions in Lesson 3.4, we used an area model to find the product. We can also use an area model to find the product of two mixed numbers.

Problem 1 Model of Your School

For a project, you are making a scale model of your school from foam board. You mark a rectangle on the board so that the rectangle's length is $1\frac{1}{2}$ feet and its width is $1\frac{1}{3}$ feet, as shown in the diagram. The area of the rectangular piece of board is the product of the rectangle's length and width, or $1\frac{1}{2} \times 1\frac{1}{3}$ feet.



A. We can add the areas of the parts to find the total area. Fill in the blanks to find the total area.

$$1+\frac{1}{3}+\frac{1}{2}+\frac{1}{6}=1+\frac{\boxed{\bullet}}{6}+\frac{\boxed{}}{6}+\frac{1}{6}=1+\frac{\boxed{}}{6}=\boxed{}$$

B. We can also write the mixed numbers $1\frac{1}{2}$ and $1\frac{1}{3}$ as improper fractions and then multiply. Fill in the blanks to find the area.

$$1\frac{1}{2}\times 1\frac{1}{3}=\frac{\boxed{\bullet}}{2}\times\frac{\boxed{}}{3}=\frac{\boxed{}}{\boxed{}}=\frac{2}{1}=2$$

It is more efficient to multiply mixed numbers by first writing them as improper fractions and then using the procedure for multiplying fractions.

Investigate Problem 1

1. Find each product.

$2\frac{2}{3} \times 1\frac{1}{3} =$

$2\frac{1}{4} \times 4\frac{2}{3} =$

$1\frac{2}{5} \times 2\frac{3}{4} =$

$4\frac{1}{6} \times \frac{1}{2} =$

$2\frac{5}{8} \times 4 =$

$2\frac{1}{2} \times 3\frac{1}{5} =$

$4\frac{1}{5} \times 2\frac{1}{7} =$

$3\frac{3}{8} \times 2\frac{2}{3} =$

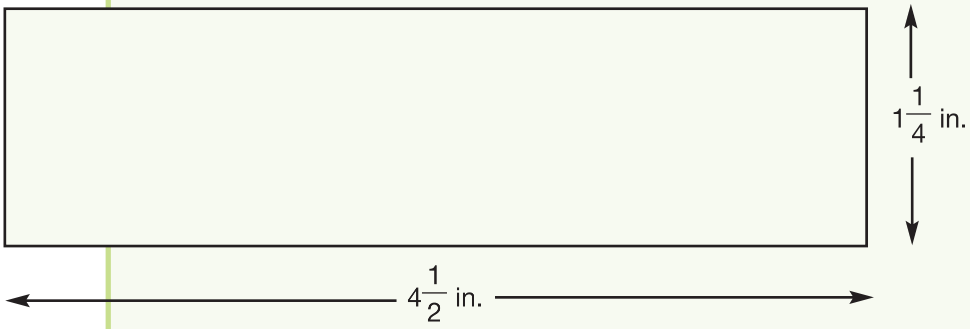
$1\frac{5}{6} \times \frac{1}{2} =$

$3 \times 2\frac{3}{4} =$

Problem 2 Making Model Desks

For the model of your school, you need to make a teacher's desk for each room. You cut the models for the tops of the desks out of balsa wood that is $1\frac{1}{4}$ inches wide. The length of the piece of balsa wood is $4\frac{1}{2}$ inches. How many model desk tops can you cut if you want each model to be $\frac{3}{4}$ inch by $1\frac{1}{4}$ inches?

A. Use your ruler to divide the wood to find the number of model desk tops that you can make.



B. Write a mixed number division problem that represents this situation.

Investigate Problem 2

1. **Math Path: Reciprocal**

Recall from Lesson 3.5 that dividing by a fraction is the same as multiplying by the multiplicative inverse, or **reciprocal,** of the fraction. We can also divide mixed numbers by first writing them as improper fractions and then multiplying by the reciprocal. Fill in the blanks to find the product.

$$1\frac{2}{3} \div 1\frac{1}{2} = \frac{\boxed{\bullet}}{3} \div \frac{\boxed{}}{2} = \frac{5}{3} \times \frac{2}{3} = \frac{\boxed{}}{\boxed{}} = 1\frac{1}{9}$$

2. Find each quotient.

$2\frac{1}{2} \div 3\frac{1}{3} =$

$2\frac{3}{4} \div 4\frac{2}{3} =$

$4\frac{1}{4} \div 2\frac{2}{3} =$

$3\frac{4}{5} \div 2\frac{3}{8} =$

$4\frac{3}{5} \div 1\frac{3}{7} =$

$4\frac{2}{3} \div 3\frac{4}{5} =$

3. Find each quotient. Then check whether your answer is reasonable by rounding each mixed number to the nearest whole number and then dividing the whole numbers.

$7\frac{2}{3} \div 1\frac{2}{3} =$

$6\frac{2}{9} \div 2\frac{7}{9} =$

$3\frac{1}{12} \div 1\frac{1}{6} =$

$8\frac{1}{8} \div 2\frac{1}{4} =$

$5\frac{3}{4} \div 2\frac{2}{3} =$

$4\frac{1}{5} \div 1\frac{5}{6} =$

3.8 Carpenter, Baker, Mechanic, and Chef

Working with Customary Units

Objectives

In this lesson, you will:

- Convert between customary units of measure.

Key Terms

- customary units of measure
- length: inch, foot, yard, mile
- capacity: fluid ounce, cup, pint, quart, gallon
- weight: ounce, pound, ton

After humankind began to own land and possessions, they needed to have a way to measure them.

Problem 1 Measure Up

At first people used body parts as measures. For instance, the length of a person's foot became the unit "one foot." The width of a person's thumb became the unit "one inch." The distance from a person's elbow to the tip of his or her finger became the unit "one cubit."

A. Turn to your partner and compare each of these "units of measure." What do you notice? Use complete sentences to describe your observations.

B. Obviously, there was a problem. People solved the problem by establishing standard units of measure that we use today, such as:

- inch, foot, yard and mile to measure distance
- ounce, pound, and ton to measure weight
- cup, pint, quart, and gallon to measure capacity

Work with your partner to complete the table that shows how the units are related.

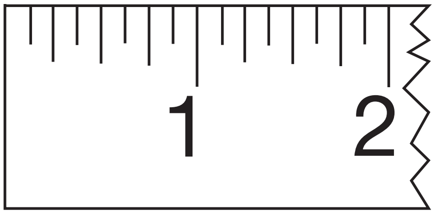
U.S. Customary Units of Measure		
Length	**Weight**	**Capacity**
1 foot =_____inches	1 pound = ____ounces	1 cup = 8 fluid ounces
1 yard =_____feet	1 ton = ______pounds	1 pint = _____cups
1 yard = 36 inches		1 quart = _____pints
1 mile = ______feet		1 gallon = _____quarts
1 mile = 1760 yards		

Investigate Problem 1

1. Math Path: U.S. Customary System of Measure

Even when units were standardized, different countries often had different standard units of measure. The units shown in the table in part (B) are part of the U.S. customary system of measure.

One problem with the U.S. customary system is that the smallest unit of measure may not be small enough. For instance, how long is an object if its length is less than one inch? The U.S. customary system relies on fractional parts of measures. In fact, an inch on a ruler is divided into several different fractional parts. Below is an inch on a ruler that has been magnified. Label the fraction of an inch that each mark represents between 0 inch and 1 inch.



2. Use a ruler to measure each line segment. Be sure to include units in your answer.

Length = ____________

Length = ____________

Length = ____________

Length = ____________

Length = ____________

Length = ____________

Problem 2A Carpenter

A carpenter is building a house and needs to cut a 12-foot long board into pieces with the following lengths:

- Three pieces that are each $8\frac{3}{4}$ inches
- Four pieces that are each $6\frac{7}{8}$ inches
- Five pieces that are each $11\frac{5}{16}$ inches

Can she cut all of these pieces from the 12-foot board? If she can, what is the length of the board that she has left over? If she cannot, which pieces can she cut so that she wastes the least amount of material? Use complete sentences to explain your reasoning. Be prepared to share your solution with the class.

Problem 2B Baker

A baker wants to make the very best bread, and he must add just the right amount of yeast by weight. He wants to make the following:

- Four batches that each use $2\frac{3}{4}$ ounces of yeast
- Ten batches that each use $4\frac{2}{3}$ ounces of yeast
- Eight batches that each use $4\frac{1}{2}$ ounces of yeast

If he can only buy yeast by the pound, will one pound be enough? Exactly how much yeast will he need? If yeast sells for $3.98 per pound, how much money will the baker spend on yeast? Will he have any yeast left over? If so, how much will he have? Use complete sentences to explain your reasoning. Be prepared to share your solution with the class.

Problem 2C Mechanic

A mechanic owns a garage. The floor of the garage cannot hold more than 24,000 pounds of weight. In the garage, the mechanic has:

- Two cars that each weigh $1\frac{6}{10}$ tons
- Three trucks that each weigh $2\frac{7}{20}$ tons
- Eight motors that each weigh $\frac{1}{8}$ ton

Has the mechanic exceeded the floor's weight limit? If so, by how many tons is he over the limit? If not, how many more $\frac{1}{8}$-ton engines could he have in the garage? Use complete sentences to explain your reasoning. Be prepared to share your solution with the class.

Problem 2D Chef

A chef needs to make several large batches of different kinds of cookies. He has the following cookies to make:

- Five batches that each require $\frac{3}{4}$ cup of vanilla extract
- Six batches that each require $\frac{2}{3}$ cup of vanilla extract
- Eight batches that each require $\frac{1}{4}$ cup of vanilla extract

How much vanilla extract will he need to buy? If he can only buy it by the quart, will one quart be enough? If not, how many quarts will he need? Use complete sentences to explain your reasoning. Be prepared to share your solution with the class.

Looking Back at Chapter 3

Key Terms

like fractions ● p. 75
unlike fractions ● p. 75
least common denominator ● p. 78
improper fractions ● p. 82
mixed number ● p. 82
U.S. customary system ● p. 85
metric system ● p. 85
remainder ● p. 89
multiplicative identity ● p. 91
multiplicative inverse ● p. 91
reciprocal ● p. 91
customary units of measure ● p. 101
length ● p. 101
inch ● p. 101
foot ● p. 101
yard ● p. 101
mile ● p. 101
capacity ● p. 101
fluid ounce ● p. 101
cup ● p. 101
pint ● p. 101
quart ● p. 101
gallon ● p. 101
weight ● p. 101
ounce ● p. 101
pound ● p. 101
ton ● p. 101

Summary

Adding Fractions with Like Denominators (p. 75)

To add fractions with like denominators, first add the numerators, then write a fraction using the sum of the numerators and the like denominator. Finally, simplify if possible.

Examples

$$\frac{1}{9}+\frac{4}{9}=\frac{1+4}{9}=\frac{5}{9}$$

$$\frac{1}{12}+\frac{3}{12}+\frac{5}{12}=\frac{1+3+5}{12}=\frac{9}{12}=\frac{3}{4}$$

Subtracting Fractions with Like Denominators (p. 75)

To subtract two fractions with like denominators, first subtract the numerators, then write a fraction using the difference of the numerators and the like denominator. Finally, simplify if possible.

Examples

$$\frac{7}{8}-\frac{2}{8}=\frac{7-2}{8}=\frac{5}{8}$$

$$\frac{11}{12}-\frac{8}{12}=\frac{11-8}{12}=\frac{3}{12}=\frac{1}{4}$$

Adding Fractions with Unlike Denominators (p. 79)

To add two fractions with unlike denominators, first find the least common denominator (LCD), then write equivalent fractions using the LCD. Next, add the resulting fractions. Finally, simplify if possible.

Examples

$$\frac{1}{4}+\frac{1}{6}=?$$

LCD: $\underline{24}$

$$\begin{array}{rl} & \dfrac{1}{4}=\dfrac{1\times 3}{4\times 3}=\dfrac{3}{12} \\ + & \dfrac{1}{6}=\dfrac{1\times 4}{6\times 4}=\dfrac{2}{12} \\ \hline & \qquad\qquad\quad =\dfrac{5}{12} \end{array}$$

$$\frac{1}{9}+\frac{5}{12}=?$$

LCD: $\underline{36}$

$$\begin{array}{rl} & \dfrac{1}{9}=\dfrac{1\times 4}{9\times 4}=\dfrac{4}{36} \\ + & \dfrac{5}{12}=\dfrac{5\times 3}{12\times 3}=\dfrac{15}{36} \\ \hline & \qquad\qquad\quad =\dfrac{19}{36} \end{array}$$

Subtracting Fractions with Unlike Denominators (p. 79)

To subtract two fractions with unlike denominators, first find the least common denominator (LCD), then write equivalent fractions using the LCD. Next, subtract the resulting fractions. Finally, simplify if possible.

Examples

$\frac{11}{15} - \frac{2}{3} = ?$

LCD:•15

$$\begin{array}{rcccc} & \frac{11}{15} = & \frac{11 \times 1}{15 \times 1} & = \frac{11}{15} \\ - & \frac{2}{3} = & \frac{2 \times 5}{3 \times 5} & = \frac{10}{15} \\ \hline & & & = \frac{1}{15} \end{array}$$

$\frac{8}{9} - \frac{1}{12} = ?$

LCD:•36

$$\begin{array}{rcccc} & \frac{8}{9} = & \frac{8 \times 4}{9 \times 4} & = \frac{32}{36} \\ - & \frac{1}{12} = & \frac{1 \times 3}{12 \times 3} & = \frac{3}{36} \\ \hline & & & = \frac{29}{36} \end{array}$$

Writing Improper Fractions as Mixed Numbers (p. 82)

To write an improper fraction as a mixed number, use long division.

Examples

$$\frac{28}{5}: \quad \begin{array}{r} 5 \\ 5\overline{)28} \\ \underline{25} \\ 3 \end{array}$$

So,•$\frac{28}{5} = 5\frac{3}{5}$.

$$\frac{37}{6}: \quad \begin{array}{r} 6 \\ 6\overline{)37} \\ \underline{36} \\ 1 \end{array}$$

So,•$\frac{37}{6} = 6\frac{1}{6}$.

Writing Mixed Numbers as Improper Fractions (p. 83)

To write a mixed number as an improper fraction, write the whole number as an equivalent fraction whose denominator is the same as the fractional part of the mixed number, then add the like fractions.

Examples

$3\frac{3}{8} = 3 + \frac{3}{8} = \frac{24}{8} + \frac{3}{8} = \frac{27}{8}$

So,•$3\frac{3}{8} = \frac{27}{8}$.

$7\frac{4}{9} = 7 + \frac{4}{9} = \frac{63}{9} + \frac{4}{9} = \frac{67}{9}$

So,•$7\frac{4}{9} = \frac{67}{9}$.

Multiplying Fractions (p. 87)

To multiply two fractions, first multiply the numerators, then multiply the denominators. Finally, simplify if possible.

Examples

$\frac{3}{7} \times \frac{5}{6} = \frac{3 \times 5}{7 \times 6} = \frac{15}{42} = \frac{5}{14}$

$\frac{11}{15} \times \frac{5}{9} = \frac{11 \times 5}{15 \times 9} = \frac{55}{135} = \frac{11}{27}$

Dividing Fractions (p. 92)

To divide two fractions, first find the reciprocal of the divisor, then multiply the dividend by the reciprocal of the divisor. Finally, simplify if possible.

Examples

$\frac{5}{6} \div \frac{3}{4} = \frac{5}{6} \times \frac{4}{3} = \frac{20}{18} = \frac{10}{9} = 1\frac{1}{9}$

$\frac{7}{8} \div \frac{2}{5} = \frac{7}{8} \times \frac{5}{2} = \frac{35}{16} = 2\frac{3}{16}$

Adding Mixed Numbers (p. 94)

To add two mixed numbers, first write the fractional parts using the LCD, then add the fractional parts. Next, add the whole number parts. Finally, simplify if possible.

Examples

$$\begin{array}{rl} & 2\frac{1}{4} = 2\frac{2}{8} \\ + & 1\frac{3}{8} = 1\frac{3}{8} \\ \hline & \quad = 3\frac{5}{8} \end{array} \qquad \begin{array}{rl} & 4\frac{5}{6} = 4\frac{15}{18} \\ + & 2\frac{1}{9} = 2\frac{2}{18} \\ \hline & \quad = 6\frac{17}{18} \end{array} \qquad \begin{array}{rl} & 7\frac{2}{3} = 7\frac{20}{30} \\ + & 3\frac{9}{10} = 3\frac{27}{30} \\ \hline & \quad = 10\frac{47}{30} = 11\frac{17}{30} \end{array}$$

Subtracting Mixed Numbers (p. 96)

To subtract two mixed numbers, first rewrite the fractional parts using the LCD. Then regroup the fractional parts if necessary and subtract the fractional parts. Next, subtract the whole number parts. Finally, simplify if possible.

Examples

$$\begin{array}{rl} & 7\frac{8}{11} = 7\frac{16}{22} \\ - & 2\frac{1}{2} = 2\frac{11}{22} \\ \hline & \quad = 5\frac{5}{22} \end{array} \qquad \begin{array}{rl} & 12\frac{5}{8} = 12\frac{5}{8} = 11\frac{13}{8} \\ - & 7\frac{3}{4} = 7\frac{6}{8} = 7\frac{6}{8} \\ \hline & \quad = 4\frac{7}{8} \end{array}$$

Multiplying Mixed Numbers (p. 97)

To multiply two mixed numbers, first write each mixed number as an improper fraction, then multiply the improper fractions. Finally, simplify if possible.

Examples

$$2\frac{6}{7} \times 3\frac{1}{4} = \frac{20}{7} \times \frac{13}{4} = \frac{260}{28} = \frac{65}{7} = 9\frac{2}{7}$$

Dividing Mixed Numbers (p. 99)

To divide two mixed numbers, first write each mixed number as an improper fraction. Then, multiply by the reciprocal. Finally, simplify if possible.

Examples

$$5\frac{3}{8} \div 3\frac{1}{4} = \frac{43}{8} \div \frac{13}{4} = \frac{43}{{}_{2}\cancel{8}} \times \frac{\cancel{4}^{1}}{13} = \frac{43}{26} = 1\frac{17}{26}$$

Converting Between Customary Units of Measure (p. 103)

To convert between customary units of measure, you need to know the values for common customary units, as shown in Lesson 3.8.

Example You are making 6 bracelets. Each bracelet uses $6\frac{1}{2}$ inches of string.

How many feet of string will you use for all 6 bracelets?

$$6 \times 6\frac{1}{2} = \frac{6}{1} \times \frac{13}{2} = \frac{78}{2} = 39;\ 39 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{39}{1}\,\cancel{\text{in}}. \times \frac{1 \text{ ft}}{12\,\cancel{\text{in}}.} = \frac{39}{12} \text{ ft} = 3\frac{1}{4} \text{ ft}$$

You will use $3\frac{1}{4}$ feet of string.