Looking Ahead to Chapter 1

1

Focus In Chapter 1, you will work with whole numbers and their operations. You will discover ways to find common multiples and common factors and determine whether a number is prime or composite. You will also learn how to tell whether your solution to a problem is reasonable.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 1.

Find the sum or difference.

1. 27 + 94 **2.** 57 – 38 **3.** 83 – 68

Find the product or quotient.

4. 8×14 **5.** 21×17 **6.** $108 \div 45$

Multiply.

7. $5 \times 5 \times 5$ **8.** $9 \times 9 \times 9 \times 9$ **9.** $3 \times 3 \times 3 \times 3 \times 3$

Read the problem scenario below.

You go to the store with $20. A notebook costs $4, a pack of pens costs $3, and a pack of pencils costs $2.

10. What is the total cost of a notebook and a pack of pens?

11. How much money do you have left after you purchase a notebook and a pack of pens?

12. Your friend goes to the same store with $25 and buys 3 notebooks. What is the total cost of 3 notebooks?

13. How much money does your friend have left after he purchases 3 notebooks?

Key Terms

expression ● p. 9
order of operations ● p. 9
variable ● p. 10
variable expression ● p. 10
factor ● p. 11
factor pair ● p. 11
Commutative Property of Multiplication ● p. 12
multiple ● p. 14
divisible ● p. 14
common multiple ● p. 17
least common multiple ● p. 17
composite number ● p. 21
prime number ● p. 21
multiplicative identity ● p. 22
prime factorization ● p. 24
factor tree ● p. 24
Associate Property of Multiplication ● p. 25
power ● p. 28
base ● p. 28
exponent ● p. 28
common factor ● p. 30
greatest common factor ● p. 30

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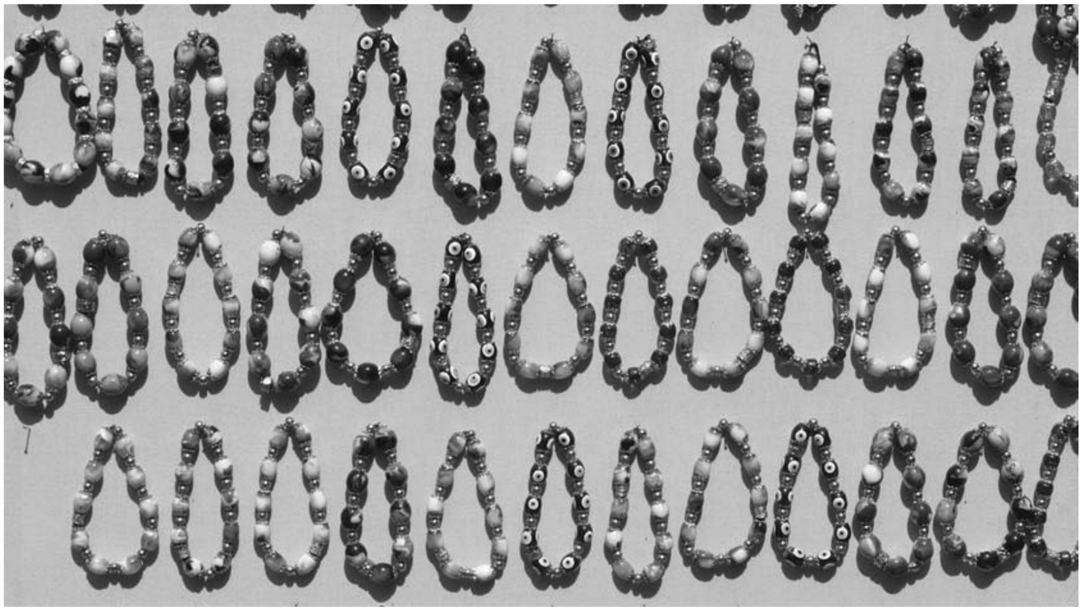
Number Sense and Algebraic Thinking



The art of attaching beads to one another to make jewelry is called beadwork. In Lesson 1.7, you will solve problems about making beaded jewelry.

Mathematical Representations

INTRODUCTION Mathematics is a human invention, developed as people encountered problems that they could not solve. For instance, when people first began to accumulate possessions, they needed to answer questions such as: How many? How many more? How many less?

People responded by developing the concepts of numbers and counting. Mathematics made a huge leap when people began using symbols to represent numbers. The first “numerals” were probably tally marks used to count weapons, livestock, or food.

As society grew more complex, people needed to answer questions such as: Who has more? How much does each person get? If there are 5 members in my family, 6 in your family, and 10 in another family, how can each person receive the same amount?

During this course, we will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand our world. The following processes can help you solve problems.

Discuss to Understand

- Read the problem carefully.
- What is the context of the problem? Do you understand it?
- What is the question that you are being asked? Does it make sense?

Think for Yourself

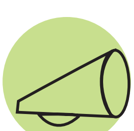
- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?

Work with Your Partner

- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to one another?

Work with Your Group

- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to one another?
- How can we explain our solution to one another? To the class?



Share with the Class

- Here is our solution and how we solved it.
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?

1.1 Money, Money, Who Gets the Money?

Introduction to Picture Algebra

Objectives

In this lesson, you will:

- Use picture algebra to represent a problem.
- Use the order of operations.

Key Terms

- order of operations

Throughout this course, you will be solving problems much like the problems that you encounter in real life.

Problem 1 Building a Business Together

You and your friends Jamal and Carla decide to make some money during summer vacation by building and selling dog houses. To get the business started, Jamal contributes \$25.55 and Carla contributes \$34.45. You all agree that each person will earn the same amount of money after Jamal and Carla get back what they invested. During the summer, your business earns a total of \$450. How much money does each person get at the end of the summer?

A. Work together as a class to complete the following tasks.

Read the problem carefully.

What is the context of the problem? Do you understand it?

What is the question that you are being asked?

B. Work by yourself to answer the following questions.

Do I need any additional information to answer the question?

Is this problem similar to some other problem that I know?

How can I represent the problem with a picture, symbols, or another representation?

You will use some of the other processes described on page 4 for Problem 1 later in the lesson. When we are solving a problem, there may be many different strategies that we can use. One useful strategy is to try to solve a simpler problem that is similar to the difficult problem. We can then use the same reasoning to solve the more difficult problem.

For example, instead of solving Problem 1, let's try to solve a simpler problem.

Problem 2 Two Boards for a Doghouse

To build a doghouse, you and your friends cut a 12-foot board into two boards. One of the boards is 4 feet longer than the other. How long is each board?

Here is an example of when a "representation" or picture can help you to solve a problem. In the space below, draw the two boards, one underneath the other. Place one end of each board against the line. Do not worry about drawing the boards exactly to scale.

Investigate Problem 2

1. Are the boards the same length? If not, which board is longer and by how much?

2. Do you know how long any part of either board is? If so, label that part. Label the unknown parts with a question mark.

3. Can you "see" a way to solve the problem? If so, write complete sentences to explain what you need to do to solve the problem. What are the lengths of the boards?

Problem 3 Fido and Jet

Fido and Jet are two small dogs. Fido weighs exactly 10 pounds more than Jet. Together they weigh exactly 46 pounds. How much does each dog weigh?

A. Use a representation similar to the one you used in Problem 2 to draw and label two "boards" that represent Fido's and Jet's weights.

B. Use the picture that you drew to help you solve the problem. How much does each dog weigh? Write your answer using a complete sentence.

Problem 1 Revisited

You and your friends Jamal and Carla decide to make some money during summer vacation by building and selling dog houses. To get the business started, Jamal contributes $25.55 and Carla contributes $34.45 to buy the equipment and materials. You all agree that each person will earn the same amount of money after Jamal and Carla get back what they invested. Your business earns a total of $450. How much money does each person get at the end of the summer?

A. Form a group with another partner team. Use a representation similar to the one you used in Problem 2 to draw and label three "boards" that represent the amounts of money that you, Jamal, and Carla will get at the end of the summer. Then share your picture with others in your group.

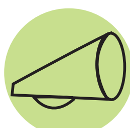
Problem 1 Revisited

Think about how you will solve the problem and what other information you may need to solve the problem. Then have each person in the group describe how he or she thought about the problem.

Use your representation to solve the problem. Describe how you solved the problem using complete sentences.

Have each group member share his or her solution with the group. Then decide whether each member's reasoning and answer makes sense.

As a group, decide how you can explain your solution to the class. Write your explanation using complete sentences.



B. Prepare a short presentation to share with your class that describes how you solved the problem. If your group could not solve the problem, explain part of your solution, and ask the class for input as to how you can complete the problem.

Could your group have used a different strategy to solve the problem? Use complete sentences to explain why or why not.

Investigate Problem 1

1. Math Path: Order of Operations

When you were solving Problem 1, you may have written an **expression** such as (450 – 25.55 – 34.45) ÷ 3. To find the value of such an expression, you need to use a set of rules called the **order of operations** so that you get the same answer as everyone else in your group. In your group, read the order of operations together.

Order of Operations

1. Evaluate expressions inside grouping symbols like ( ) or [ ].
2. Multiply and divide from left to right.
3. Add and subtract from left to right.

2. Use the order of operations to determine whether the values of each pair of expressions are equal. The first problem was completed for you.

$(4 + 2) \times 5$ and $(111 - 21) \div 3$

$(4 + 2) \times 5$	$(111 - 21) \div 3$
6×5	$90 \div 3$
30	30

The expressions are equal.

$17 \times (5 + 1)$ and $(13 + 21) \times 3$

$5 + 6 \times 12$ and $126 \div 2 - 1$

3. For each expression, decide where to place parentheses so that the answer is correct using the order of operations.

$27 \div 3 + 6 + 1$	$6 \times 15 - 12 + 3$
Answer: 4	Answer: 21

Did You Know?

Variables are used to represent numbers.

1

1. Your friend has a guppy fish who just had babies. There are 9 baby guppies. Your friend gives several baby guppies to you. You can represent this situation as $9 - \boxed{?}$.

If your friend gives you 5 baby guppies, how many baby guppies are left? Write a complete sentence to explain how you found your answer.

Often, mathematicians use letters as placeholders when a value is not known. The letter is called a *variable.* A **variable** is a symbol used to represent a value. A **variable expression** consists of numbers, variables, and operations.

2. You manage a clothing store. The store has 9 mannequins to display outfits. Each year, you plan to purchase additional mannequins. A variable expression to represent the situation is

$9 + m$

where m is the number of mannequins you will purchase. Write the expression that represents the total number of mannequins if you purchase 2 mannequins. How many mannequins will you have?

Write the expression that represents the total number of mannequins if you purchase 4 mannequins. How many mannequins will you have?

3. When writing the product of a number and a variable, you do not need to use a multiplication symbol. So, the expression $5 \times y$ is the same as $5y$.

Find the value of the expression when $a = 16$.

$24 - a$ $\qquad$ $9 + a$ $\qquad$ $\frac{a}{4}$ $\qquad$ $3a$

4. When solving a real-life problem, you can write the important information in mathematical terms. For example, "you have 10 fewer CDs than Mary" can be written as $m - 10$, where m represents the number of CDs that Mary has. Write each phrase as a variable expression. Let n represent the number.

20 increased by a number ____________

the product of 6 and a number ____________

a number subtracted from 31 ____________

1.2 Collection Connection

Factors and Multiples

Objectives

In this lesson, you will:

- List factor pairs of numbers.
- Relate factors, multiples, and divisibility.

Key Terms

- factor
- factor pair
- Commutative Property of Multiplication
- multiple
- divisible

Take Note

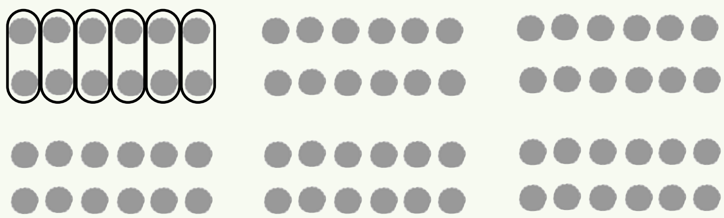
When you multiply two numbers to produce another number, each number that you multiply is a **factor** of the resulting number (or product). For instance, 2 is a factor of 12.

As people began to collect possessions, they needed ways to not only count them but to also group them for different reasons. Can you think of some reasons why people may want to group their possessions?

Problem 1 Factors of 12

Your uncle has a bottle cap collection. He wants to display 12 bottle caps in each box.

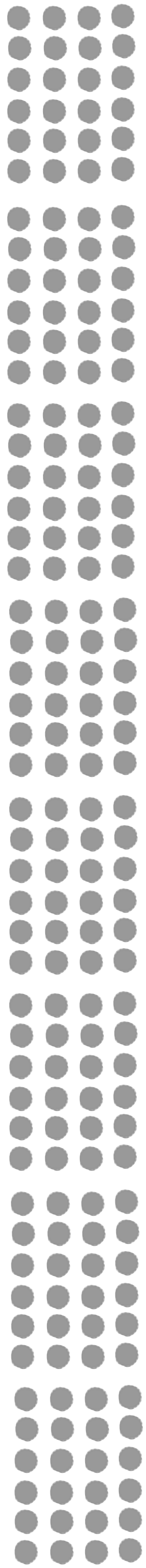
A. In each box below, collect the caps into smaller groups so that there are the same number of caps in each group. Draw a circle around each group. Then record the numbers of groups and caps in the table below. Repeat this process for each box, using a different number of groups each time. The first box is already done for you.



B. Complete the third column of the table by multiplying the number of groups by the number of caps in each group.

C. A **factor pair** is two numbers that are multiplied together to produce another number. For instance, one factor pair for the number 12 is 2×6. Complete the table by writing the factor pair for each grouping. Record your results in the last column of the table.

Number of Groups	Number of Caps in Each Group	Number of Groups Multiplied by Number of Caps in Each Group	Product Written as Factor Pair
6	2	12	6×2



Investigate Problem 1

1. List all of the unique factor pairs from your table. Is the total number of different groupings the same as the number of unique factor pairs?

2. **Math Path: Commutative Property of Multiplication**

   In your diagram, 6 groups of 2 items looks different from 2 groups of 6 items. When you multiply the factor pairs for these groupings, though, you get the same number. This means that $6 \times 2 = 2 \times 6$.

   This is an example of a very important property of numbers, the **Commutative Property of Multiplication.** You will learn more about this property in Chapter 14. Use your factor pairs to write two other examples of the Commutative Property of Multiplication.

3. Use the factor pairs listed in your table to list all of the unique factors of 12.

Problem 2 Factors of 24

Complete the table for 24 bottle caps. Use the pictures at the left if needed.

Number of Groups	Number of Caps in Each Group	Number of Groups Multiplied by Number of Caps in Each Group	Product Written as Factor Pair

Investigate Problem 2

1. List all of the unique factors of 24.

2. In your own words, write definitions for the following terms.

   factor:

   factor pair:

3. In Problems 1 and 2, you found the unique factors of 12 and 24 by dividing a collection of items into groups of equal size. Work together with your partner to think of another way to find the factors of a number. Then use complete sentences to write the steps for your method below.

4. Exchange the steps of your method with another pair of partners. Have them follow your directions to see if they can find the unique factors of 30. At the same time, follow their methods to find the unique factors of 30.

5. Use the method of your choice to find all of the unique factors of each number listed in the table.

Number	Unique Factors
7	
25	
31	
36	
44	
48	

Explain how you found the unique factors of 48. Use complete sentences in your answer.

1

Problem 3 Getting Things Straight

A. Hot dogs are normally sold only in packages of 10. If you buy 1 package, how many hot dogs do you have?

If you buy 2 packages?

If you buy 3 packages?

If you buy 4 packages?

B. In each case, you multiply by 10 to get the total number of hot dogs. So, the total number of hot dogs is a **multiple** of what number? Write complete sentences to explain your answer.

Investigate Problem 3

1. In your own words write a definition for multiple.

multiple:

2. Find the first five multiples of each number in the table.

Number	First Five Multiples
7	
25	
31	
36	
44	
48	

3. **Math Path: Factors, Multiples, Divisibility**

In mathematics, there are many ways to show the relationship between a number and one of its factors. For example, we can say that:

6 is a factor of 12.
12 is a multiple of 6.
12 is divisible by 6.

Use complete sentences to explain why 12 is a multiple of 6.

Use complete sentences to explain why 12 is divisible by 6.

Take Note

A number is **divisible** by another number if the quotient of the first number and the second number has a remainder of 0. So, 36 is divisible by 9 because $36 \div 9 = 4$ with a remainder of 0.

1.3 Dogs and Buns

Least Common Multiple

Objectives

In this lesson, you will:

- Find the least common multiple of two numbers.

Key Terms

- common multiple
- least common multiple

Problem 1 Waste Not, Want Not

Your club wants to raise money at the Spring Festival by selling hot dogs. Juan visits the supermarket to determine the cost of the hot dogs and buns. When he returns, he reports that the club may have a small problem. He found that a package of 10 hot dogs costs $2.39 and a package of one dozen buns costs $1.99.

Because the club wants to spend as little money as possible, Juan is worried that no matter how many packages of each they buy, the club will have wasted either some hot dogs or buns.

A. Is Juan correct? Use a complete sentence to explain why or why not.

B. Work with your group to answer the question above. Determine the number of packages of hot dogs and buns that the club should buy in order to have the least amount of waste. Remember that sometimes it helps to draw a picture to "see" the problem. Use a table to record your results.

Diagram: Table:

Investigate Problem 1

1. Share your work and your answer to the question with another group. Does each group have the same answer? Be sure that each group understands how the other group determined their answer.

2. As a group, present your solution to the entire class.

3. The club wants to have enough hot dogs and buns to serve 180 people. How many packages of each should the club buy, assuming that each person only eats one hot dog and one bun? What is the total cost? Use complete sentences to answer the questions.

4. How does the total number of hot dogs increase when you purchase another package? How does the total number of buns increase when you purchase another package? Why? Use complete sentences to explain.

5. What term describes the total number of hot dogs or buns in relation to the number of hot dogs or buns in each package?

Problem 2 Multiples and More

In the hot dogs and buns problem, the total number of hot dogs is a multiple of 10 and the total number of buns is a multiple of 12.

A. List the first 10 multiples of each number.

Multiples of 10:

Multiples of 12:

B. Examine both lists of multiples. What do you "see" that would have helped you determine the number of hot dogs and buns to buy so that none were wasted? Write your answer using a complete sentence.

Investigate Problem 2

1. **Math Path: Common Multiple**

   When a multiple of one number is also a multiple of another number, the multiple is a **common multiple** of the numbers. For the numbers 10 and 12, list the first three common multiples.

   Common multiples of 10 and 12:

2. What do you notice about the common multiples of 10 and 12? Use complete sentences to explain.

3. Which of these common multiples was the number of hot dogs and buns to buy so that none were wasted?

4. **Math Path: Least Common Multiple**

   The smallest of the common multiples is called the **least common multiple** (or LCM). For each pair of numbers, find the least common multiple and at least one other common multiple. Later in the course, we will revisit finding the LCM of larger numbers.

   6 and 8   5 and 7   6 and 12

   9 and 4   15 and 9   11 and 6

5. Choose the word that makes the following statement true. Then use complete sentences to explain your choice.

   The LCM of two numbers is *(always, sometimes, never)* the product of the two numbers.

Do You Remember?

Rules for divisibility

1

A number is **divisible** by another number if the quotient of the first number and the second number has a remainder of 0.

Divisibility Rules

A whole number is divisible by:

2 if the number is even.	32 ÷ 2 = 16 32 is divisible by 2.	75 ÷ 2 = 37 R 1 75 is not divisible by 2.
3 if the sum of the digits in the number is divisible by 3.	81 ÷ 3 = 27 81 is divisible by 3.	85 ÷ 3 = 28 R 1 85 is not divisible by 3.
4 if the last two digits of the number are divisible by 4.	124 ÷ 4 = 31 124 is divisible by 4.	130 ÷ 4 = 32 R 2 130 is not divisible by 4.
5 if the last digit of the number is 0 or 5.	220 ÷ 5 = 44 220 is divisible by 5.	104 ÷ 5 = 20 R 4 104 is not divisible by 5.
6 if the number is divisible by both 2 and 3.	1842 ÷ 6 = 307 1842 is divisible by 6.	1664 ÷ 6 = 277 R 2 1664 is not divisible by 6.
8 if the last three digits of the number are divisible by 8.	33,112 ÷ 8 = 4139 33,112 is divisible by 8.	17,309 ÷ 8 = 2163 R 5 17,309 is not divisible by 8.
9 if the sum of the digits in the number is divisible by 9.	963 ÷ 9 = 107 963 is divisible by 9.	824 ÷ 9 = 91 R 5 824 is not divisible by 9.
10 if the last digit of the number is 0.	240 ÷ 10 = 24 240 is divisible by 10.	368 ÷ 10 = 36 R 8 368 is not divisible by 10.

Tell whether each number is divisible by the given number.

789

divisible by 2? _______

divisible by 3? _______

divisible by 4? _______

divisible by 6? _______

345

divisible by 2? _______

divisible by 3? _______

divisible by 5? _______

divisible by 9? _______

5614

divisible by 2? _______

divisible by 4? _______

divisible by 8? _______

divisible by 10? _______

416

divisible by 2? _______

divisible by 4? _______

divisible by 8? _______

divisible by 10? _______

8490

divisible by 2? _______

divisible by 3? _______

divisible by 6? _______

divisible by 10? _______

10,398

divisible by 2? _______

divisible by 3? _______

divisible by 4? _______

divisible by 6? ______

1.4 Kings and Mathematicians

Prime and Composite Numbers

Objectives

In this lesson, you will:

- Understand prime and composite numbers.
- Become familiar with the multiplicative identity.

Key Terms

- prime number
- composite number
- multiplicative identity

Problem 1 One Hundred Boxed Gifts

A king is given 100 gifts in 100 boxes by a mathematician. All of the gifts are the same size and are lined up on a long table in a single row. The mathematician tells the king that he only gets to keep the gifts if he follows the mathematician's instructions and can then answer two questions. If the king cannot answer the mathematician's questions correctly, he must give up his throne to the mathematician.

The mathematician gives the king the following instructions:

- All of the gifts are numbered from 1 to 100.
- The king cannot open the first or second gift.
- The king must open every second gift after gift Number 2 (every other gift).
- The king must then go to the next unopened gift, gift Number 3. He must not open it, but must open every third gift after gift Number 3.
- The king must go to the next unopened gift, gift Number 5. He must not open it, but must open every fifth gift after gift Number 5.
- The king must continue in this fashion until he gets to the last gift.

The mathematician asks the king to look at the number on each of the gifts that are not opened. The mathematician then asks the two important questions:

Ignoring the Number 1, in what way are all of the numbers on the unopened gifts the same and in what way are they different from the numbers on the opened gifts?

If there were more gifts, what are the next three gifts that would not have been opened?

To solve this problem, draw a diagram to represent the first 20 gifts. Then use the diagram to follow the mathematician's instructions.

Investigate Problem 1

1. List the numbers of the unopened gifts.

2. List the numbers of the opened gifts.

3. Answer the first of the mathematician's questions:

   Ignoring the Number 1, in what way are all of the numbers on the unopened gifts the same and in what way are they different from the numbers on the opened gifts?

   If you and your partner are having problems, extend your diagram to represent 30 or 40 gifts.

4. Share your answers with another pair of partners. Then work together with that team to prepare an answer to the second of the mathematician's questions:

   If there were more gifts, what are the next three gifts that would not have been opened?

In the One Hundred Boxed Gifts problem, the number on each of the unopened gifts has only two factors, the number itself and 1. Numbers greater than 1 with exactly two whole number factors are called **prime numbers**. Numbers that have more than two whole number factors are called **composite numbers**. Only the number 1 has a single factor, so the number 1 is neither a prime nor a composite, which makes the number 1 very special.

Problem 2 *Sieve of Eratosthenes*

The One Hundred Boxed Gifts problem is very similar to a method for finding prime numbers first discovered by Greek mathematician Eratosthenes over 4000 years ago. He called it the Sieve. A sieve was a tool that was used to separate small particles from larger particles and was usually a box with a screen for a bottom so that the smaller pieces could fall through.

The Sieve of Eratosthenes screens out all of the composite numbers and leaves only the primes. Let's use it to find all of the primes up to 100. Below are the first 100 numbers written in order in an array, which is another representation!

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A. Start by putting a square around the number 1 because it is neither prime nor composite.

B. Circle the number 2 and cross out all of the multiples of 2.

C. Circle the next number after 2 that is not crossed out. Then cross out its multiples that are not already crossed out.

D. Continue in this fashion until you come to the first number greater than 10 that is not crossed out. All of the remaining numbers have "fallen through the sieve" and are prime numbers.

Investigate Problem 2

1. How many of the prime numbers are even? Use complete sentences to explain your answer.

2. Is it possible that there is an even prime greater than 100? Use complete sentences to explain why or why not.

3. Explain why you need to continue crossing out numbers only until you come to the first number greater than 10 that is not crossed out. Write your answer using a complete sentence.

4. How do you know that any remaining number less than 100 must be a prime number without continuing to use the sieve process? Use complete sentences to explain your reasoning.

5. **Math Path: Multiplicative Identity**

   The number 1, besides being neither prime nor composite, is also the only number that is a factor of every number. The number 1 has the special property that when it is multiplied by any number, the product is that number. Because of this property, the number 1 is called the **multiplicative identity.**

   You have learned two properties of mathematics so far in this course. Identify the property that is shown by each example.

   $21 \times 15 = 15 \times 21$ $\qquad$ $45 \times 1 = 45$

1.5

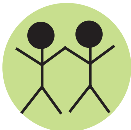
I Scream for Ice Cream

Prime Factorization

Objectives

In this lesson, you will:

- Find the prime factorization of a number.
- Use the associative property of multiplication.



Key Terms

- prime factorization
- factor tree
- Associative Property of Multiplication

Problem 1 Ice Cream Containers

Your school plans to make and sell homemade ice cream. They are having a contest for the container design. One of the design requirements is that the length, width, and height be whole numbers greater than 1. The container must be designed to hold about 1 gallon of ice cream. One gallon of ice cream takes up 210 cubic inches of space. What are the possible dimensions of the container?

A. The volume of the container must be 210 cubic inches. To find the volume of the container, multiply the container's length by its width and height:

Volume = length × width × height

Complete the table by listing three possible dimensions of the container.

Length (inches)	Width (inches)	Height (inches)	Volume (cubic inches)
			210
			210
			210

B. For each row in the table, write the volume as the product of prime numbers. Use a complete sentence to describe what you observe in each case.

Investigate Problem 1

1. Math Path: Prime Factorization

Recall that a composite number is a number that has more than two whole number factors. It turns out that every composite number can be written as the product of prime numbers. Writing a whole number as the product of prime numbers is the **prime factorization** of the number.

For example, 4 is the smallest composite number. Write 4 as the product of prime numbers.

$4 = \_\_\_\_ \times \_\_\_\_$

2. Write each composite number as the product of primes.

6 = _________ 8 = _________ 9 = _________

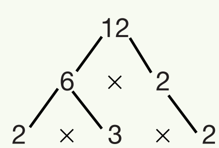
10 = _________ 12 = _________ 14 = _________

Problem 2 Factor Trees

A. It may be easy to write most small numbers as products of primes. For larger numbers, it may be more difficult. Write 144 as the product of primes.

144 = ____________________

B. One organized representation that can help you to find the prime factorization quickly is a factor tree. A **factor tree** for 12 is shown below.



Use the steps to write a factor tree for 30. Then write the prime factorization of 30.

Begin by writing 30 at the top.

Pick any pair of whole number factors of 30 other than 1 and 30. Draw a branch from 30 to each factor.

If both of the factors are prime, then you are finished. If not, use branches to write a factor pair for any composite factors.

Continue to find factor pairs until all of the factors of 30 are prime.

Use the factor tree to write the prime factorization of 30.

30 = __________________________

Investigate Problem 2

1. Is the factor tree in Problem 2 the only factor tree that you could write for 30? If not, find a least one more factor tree for 30.

30

______ × ______

Take Note

Remember that a product is the name given to the answer in a multiplication problem.

2. Check with you partner to see if he or she came up with the same factor trees for 30 as you. How many different factor trees are there for 30?

3. **Math Path: Associative Property of Multiplication**

   Notice that you can group the prime factors together in any order and multiply them to get the same product. For example,

   $(2 \times 3) \times 2 = 12$ and $2 \times (3 \times 2) = 12$.

   This is an example of another important property of numbers, the **Associative Property of Multiplication.** You will learn more about this property in Chapter 14.

   Use the prime factorization of 30 to write an example of the Associative Property of Multiplication.

Take Note

Be sure to follow the order of operations to find the value of an expression. Remember that when parentheses are used in an expression, always perform the operations inside the parentheses first.

4. Work with your partner to construct a factor tree for each number. Then write the prime factorization of each number.

   24

   81

   96

1

Investigate Problem 2

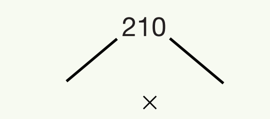
5. Share your factor trees with another pair of partners. For each number, are your factor trees the same as or different from the factor trees of the other pair of partners?

6. For each number, is your prime factorization the same as or different from the prime factorization of the other partner team? Use a complete sentence to answer the question.

Problem 1 Revisited

Your school plans to make and sell homemade ice cream. They are having a contest for the container design. One of the design requirements is that the length, width, and height be whole numbers greater than 1. The container must be designed to hold about 1 gallon of ice cream. One gallon of ice cream takes up 210 cubic inches of space. What are the possible dimensions of the container?

A. Construct a factor tree for the number 210. Then write the prime factorization of 210.



B. Use the prime factorization to list all of the possible dimensions of the container in the table below.

Length (inches)	Width (inches)	Height (inches)	Volume (cubic inches)
			210
			210
			210
			210
			210
			210

1.6

Powers That Be

Powers and Exponents

Objectives

In this lesson, you will:

- Use powers and exponents to write repeated multiplication.
- Use powers and exponents to write the prime factorization of a number.

Key Terms

- power
- base
- exponent

Problem 1 Prime Factorization

A. Use a factor tree to find the prime factorization of 64.

64

The prime factorization of 64 is ____________________.

B. Use a factor tree to find the prime factorization of 81.

81

The prime factorization of 81 is _______________.

C. What do you notice about the prime factorizations of 64 and 81? Use a complete sentence to answer the question.

Once again, think about how mathematics was discovered. After human beings began counting their possessions, they needed to "put groups together" and "take from groups," so the operations of addition and subtraction were invented.

Much later people formed communities and began to trade goods. At this point, they needed to add the same number repeatedly. For instance, a blacksmith who made horseshoes needed to know how many shoes he would need for all of the king's 125 horses. The blacksmith could have added the number 4 repeatedly 125 times, but by multiplying 4×125, he saved a lot of time.

1

Problem 2 Horse Sense

In the same way, suppose the blacksmith needed to shoe the horses of 4 kings. Each king had 4 bands of 4 men. Each man rode a horse (with 4 feet, of course). Each foot needed a horseshoe that took 4 nails. For the blacksmith to figure out whether he had enough nails to do the job, he could use repeated multiplication.

4 kings × 4 bands × 4 men with horses × 4 horseshoes × 4 nails

How many nails did the blacksmith need? Use a complete sentence to explain how you found this number.

Investigate Problem 2

Take Note

In a power, when no exponent is written, you can assume that the exponent is 1.

$4 = 4^1$

1. Math Path: Powers

Eventually, people devised a notation to represent repeated multiplication. This notation is called a **power.** Let's use a power to represent the number of nails that the blacksmith needs. The expression 4 × 4 × 4 × 4 × 4 has only one factor, 4, which is repeated 5 times. You can express this expression as a power.

base → 4 ← exponent

$4^5 = 4 \times 4 \times 4 \times 4 \times 4$

(4^5: power)

The **base** of a power is the factor and the **exponent** of the power is the number of times that the factor is repeated. The power can be read as:

4 to the fifth power
the fifth power of 4
4 raised to the fifth power

2. You can use powers to write the prime factorization of a number. For instance, the prime factorization of 64 is

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$.

Write the prime factorization of 81 using a power. How would you read this?

81 =

Take Note

When you write the prime factorization of a number, write the factors in increasing order. For instance,

$36 = 2^2 \times 3^2$.

3. Write the prime factorization of each number using powers. The first one is done for you.

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$ 45 = ____________

144 = ____________ 256 = ____________

343 = ____________ 625 = ____________

1.7 Beads and Baubles

Greatest Common Factor

Objectives

In this lesson, you will:

- Find the greatest common factor of two or more numbers.

Key Terms

- common factor
- greatest common factor

Problem 1 Bags of Beads

Your aunt belongs to a club that makes beaded jewelry. The club wants to sell small packages of different types of beads to people who want to make their own jewelry. They plan to buy and sell three different types of beads—spacer beads, round beads, and rectangular beads. They can buy each type in bags of different quantities. The table below lists the type of bead, the quantity in each bag, and the price of a bag.

Type of Bead	Quantity per Bag	Cost per Bag
Spacer beads	40	$3.50
Round beads	72	$5.00
Rectangular beads	24	$7.50

The club members want to divide the beads into packages so that each package has exactly the same number of spacer beads, round beads, and rectangular beads. They also want to make sure that they use all of the beads they buy with no beads left over.

The club members are having trouble determining the greatest number of packages they can make so that all of the beads are used and there is the same number of each type of bead in each package. Can you help?

A. Do you understand the problem? Is there anything that is unclear about the problem?

B. Think about the problem. Do you have all of the information that you need? What information do you know? How do you think you should use this information to solve the problem?

C. Talk with your partner about how each of you plans to solve the problem. Then use the plans together to find the solution.

Problem 1 Bags of Beads

D. When you and your partner find a solution, form a group with another partner team. In your group, try to agree on a common solution. Be sure that everyone in the group understands the solution and can explain how the solution was found.

E. Share your solution and your solution method with the entire class.

Investigate Problem 1

1. Based on your solution, how much money will the club spend on the beads that will be put into the packages? Use a complete sentence to answer the question.

2. **Math Path: Greatest Common Factor**

In Problem 1, you found the greatest number of packages that the club could make from three different types of beads. In other words, you were looking for the greatest number that could divide evenly into three other numbers (40, 72, and 24).

In Lessons 1.2 and 1.5, you found the factors of a number and the prime factorization of a number. We will first use these ideas to identify *common factors* of two or more numbers and then to find the *greatest common factor* of the numbers.

A **common factor** is a whole number that is a factor of two or more numbers. The **greatest common factor** (or GCF) is the greatest whole number that is a common factor of two or more numbers.

Work with your partner to complete each table. First, for each number, list the unique factor pairs and the unique factors. Then look at the unique factors for both numbers to determine the common factors.

Number	Unique Factor Pairs	Unique Factors	Common Factors
12			
18			

Take Note

Remember, when you multiply two numbers to produce another number, each number that you multiply is a factor.

When you write a whole number as the product of prime numbers, you are writing the prime factorization of the number.

Investigate Problem 1

1

Number	Unique Factor Pairs	Unique Factors	Common Factors
20			
36			

Number	Unique Factor Pairs	Unique Factors	Common Factors
56			
70			

3. In each table in Question 2, circle the greatest common factor for each pair of numbers. What do you observe as the numbers get larger? Use a complete sentence to answer the question.

4. Find the greatest common factor of three numbers.

Number	Unique Factor Pairs	Unique Factors	Common Factors
24			
32			
42			

Number	Unique Factor Pairs	Unique Factors	Common Factors
64			
96			
128			

Investigate Problem 1

5. As the numbers increase, finding all of the factors and the GCF becomes more difficult and time consuming. However, there is a more efficient way to find the GCF using prime factorizations.

Let's find the prime factorizations of 64, 96, and 128, and use the results to find the GCF of the numbers.

Complete the table below by first finding the prime factorization of each number. Then list all of the common prime factors. Be sure that if a prime number is a factor of all three numbers more than once, you list it the number of times it appears. The greatest common factor is the product of the common prime factors.

Number	Prime Factorization	Common Prime Factors	Greatest Common Factor
64			
96			
128			

Was the greatest common factor the same using this method as the method that you used in Question 4?

6. Find the greatest common factor of each set of numbers by using prime factorizations.

Number	Prime Factorization	Common Prime Factors	Greatest Common Factor
54			
45			
72			

Number	Prime Factorization	Common Prime Factors	Greatest Common Factor
144			
180			
96			

Looking Back at Chapter 1

Key Terms

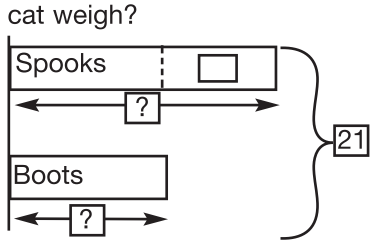
- expression • p. 9
- order of operations • p. 9
- variable • p. 10
- variable expression • p. 10
- factor • p. 11
- factor pair • p. 11
- Commutative Property of Multiplication • p. 12
- multiple • p. 14
- divisible • p. 14
- common multiple • p. 17
- least common multiple • p. 17
- composite number • p. 21
- prime number • p. 21
- multiplicative identity • p. 22
- prime factorization • p. 24
- factor tree • p. 24
- Associate Property of Multiplication • p. 25
- power • p. 28
- base • p. 28
- exponent • p. 28
- common factor • p. 30
- greatest common factor • p. 30

Summary

Using Picture Algebra (p. 6)

You can use a picture or diagram to help you solve word problems. In your diagram, represent the known values from the problem and represent the unknown values using a question mark. Then solve the problem.

Example Spooks and Boots are two cats. Spooks weighs exactly 9 pounds more than Boots. Together, the cats weigh 21 pounds. How much does each cat weigh?



From the picture, you can see that Boots weighs 6 pounds and Spooks weighs 6 + 9, or 15 pounds.

Using the Order of Operations (p. 9)

To use the order of operations, first evaluate expressions inside grouping symbols, then multiply and divide from left to right. Finally add and subtract from left to right.

Examples You can use the order of operations to determine whether the values of each pair of expressions are equal.

$27 \div (4 + 5)$ and	$(127 - 94) \div 3$	$4 \times (45 - 7)$ and	$(18 + 58) \times 2$
$27 \div 9$	$33 \div 3$	4×38	76×2
3	11	152	152

The expressions are not equal. The expressions are equal.

Finding Factors (p. 12)

Example List the unique factors of 32.

1×32 2×16 4×8

The factors of 32 are 1, 2, 4, 8, 16, and 32.

1

Using the Commutative Property of Multiplication (p. 12)

The Commutative Property of Multiplication states that you can multiply two factors in any order. So, for any numbers a and b, then $a \times b = b \times a$.

Example The Commutative Property of Multiplication states that the values in the expressions below are equal.

4×21 and 21×4
84 84

The expressions are equal, so $4 \times 21 = 21 \times 4$.

Finding Multiples (p. 14)

Example List the first 5 multiples of 8.

$8 \times 1 = 8$ $8 \times 2 = 16$ $8 \times 3 = 24$ $8 \times 4 = 32$ $8 \times 5 = 40$

The first 5 multiples of 8 are 8, 16, 24, 32, and 40.

Finding Common Multiples (p. 17)

To find a common multiple of two or more numbers, list several multiples of each number. Then find the multiples the numbers have in common.

Example The common multiples of 4 and 10 are:

Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, 44, 48, 52, 56, **60**
Multiples of 10: 10, **20**, 30, **40**, 50, **60**

Three common multiples of 4 and 10 are 20, 40, and 60.

Finding the Least Common Multiple (p. 17)

To find the least common multiple of two or more numbers, find the smallest of the common multiples.

Example The common multiples of 5 and 12 are:

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, **60**, 65, 70, 75, 80, 85
Multiples of 12: 12, 24, 36, 48, **60**, 72, 84, 96, 108, 120

The least common multiple of 5 and 12 is 60.

Finding Prime and Composite Numbers (p. 21)

To determine whether a number is prime or composite, find the whole number factors of the number. If the only factors are 1 and the number, then the number is prime.

Examples The only factors of 17 are 1 and 17. So, 17 is prime.

The factors of 39 are 1, 3, 13, and 39. So, 39 is composite.

Use the Multiplicative Identity (p. 22)

The multiplicative identity states that when the number 1 is multiplied by any number, the result is that number. So, for any number a, $a \times 1 = a$.

Examples The multiplicative identity tells you that the following expressions are true.

$35 \times 1 = 35$ $1 \times 121 = 121$ $d \times 1 = d$

Writing the Prime Factorization (p. 24)

Example To write the prime factorization of a number, use a factor tree.

$$
\begin{array}{c}
45 \\
/\ \backslash \\
9 \times 5 \\
/\backslash\ \ \backslash \\
3 \times 3 \times 5
\end{array}
$$

The prime factorization of 45 is $3 \times 3 \times 5$.

Using the Associative Property of Multiplication (p. 25)

The Associative Property of Multiplication states that you can group factors in any order and multiply them to get the same product. So, for any numbers *a, b,* and *c,* $(a \times b) \times c = a \times (b \times c)$.

Examples The Associative Property of Multiplication states that the values of each pair of expressions are equal.

$(5 \times 31) \times 16$	and	$5 \times (31 \times 16)$	$9 \times (32 \times 17)$	and	$(9 \times 32) \times 17$
155×16	and	5×496	9×544	and	288×17
2480		2480	4896		4896

The expressions are equal. The expressions are equal.

Writing Powers (p. 28)

To express repeated multiplication as a power, write the base raised to the number of times the base is multiplied by itself.

Example The expression $7 \times 7 \times 7 \times 7 \times 7$ is equal to the power 7^5. You read the power as "seven raised to the fifth power."

Writing the Prime Factorization Using Powers (p. 28)

To write the prime factorization of a number using powers, find the prime factorization. Then rewrite the prime factorization using powers.

Example $54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

The prime factorization of 54 is 2×3^3.

Finding the Greatest Common Factor (p. 30)

To find the greatest common factor of two or more numbers, first write the prime factorization of each number. Then list the common prime factors. Be sure that if a prime number is a factor of all of the numbers more than once, you list the number the number of times it appears. The greatest common factor is the product of the common prime factors.

Example $36 = 2^2 \times 3^2$ $48 = 2^4 \times 3$ $64 = 2^6$

The common prime factors of 36, 48, and 64 are 2×2, or 2^2. So, the greatest common factor of 36, 48, and 64 is $2^2 = 4$.