

## Problem Solving for Irish Second level Mathematicians

THURSDAY 19TH OCTOBER 2006

### *Senior Cycle*

Time allowed: **60 minutes**

#### **Rules and Guidelines for Contestants**

1. The candidate number (on the top right of your answer sheet) is the number you must have in order to get your result. Write this number down somewhere and keep it safe. **You must know this number in order to get your result.**
2. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).
3. **Use a pencil to fill out the answer sheet.** B or HB pencils are the best for this. If you make a mistake, you can erase the error and correct it.
4. Write your name clearly (in block capitals) in the space provided in the answer sheet.
5. You should have some extra sheets of your own paper (or a refill pad) for rough work while you are doing the questions. **Do not** hand these up at the end.
6. When you have decided on your answer for a particular question, carefully mark your choice for that question in the way shown on the answer sheet - with a horizontal line through your choice. An X or a tick mark will not count.
7. Remember that the marking will be done by a machine. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions. **Any scribbles or other marks could mean that your answer sheet will not be marked.** Also, **do not fold or bend the answer sheet.**
8. Some of the questions are quite difficult, and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answer to all of the questions.  
The questions at the beginning are easier than those at the end.  
The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions - take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.
9. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

*Good luck and thank you for participating in PRISM.  
We hope you will enjoy the problems!*

1. Which one of the following numbers is **not** equal to 2?

- (A) The number of hind legs of a dog.
- (B) The number of wings of a blackbird.
- (C) The number of teams that contested the 2006 All-Ireland Senior hurling final.
- (D) The number of sides of a square.
- (E) The number of hours between 11p.m. on October 19 and 1a.m. on October 20 (in the same year).

2. Which one of the following numbers is equal to  $\frac{1}{2} - \frac{1}{3} + \frac{1}{4}$ ?

- (A)  $\frac{1}{9}$       (B)  $\frac{5}{12}$       (C) 1      (D)  $\frac{12}{5}$       (E)  $\frac{1}{5}$

3. Which one of the following numbers is the largest?

- (A)  $\frac{9}{10}$       (B)  $\frac{17}{20}$       (C)  $\frac{4}{5}$       (D)  $\frac{13}{15}$       (E)  $\frac{3}{4}$

4. Sammy is a snake of a strange (and fictitious) species. He grows so quickly that every day his length doubles. If this process continues indefinitely and Sammy is 12cm long now, then which of the following is the best approximation to the length he will be exactly six days from now?

- (A) The distance from Galway to Dublin.
- (B) The length of a golf club.
- (C) The length of a football field.
- (D) The length of the Earth's equator.
- (E) The width of a tennis court.

5. Which of the following numbers is the smallest?

- (A)  $2^3$       (B)  $\frac{15}{2}$       (C)  $\sqrt{90}$       (D)  $\frac{31}{4}$       (E)  $3^2$

6. John looked at a picture of a man, and truthfully said "Brothers and sisters I have none, but that man's father is my father's son."

What is the relationship between John and the man in the picture?

- (A) John is a grandfather of the man in the picture.
- (B) John is the son of the man in the picture.
- (C) John is the father of the man in the picture.
- (D) John is a grandson of the man in the picture.
- (E) John is the man in the picture.

7. If  $a + b = 10$  then what is  $a^2 + b^2 + 2a + 2b + 2ab$ ?

- (A) 25      (B) 120      (C) 101      (D) 200      (E) 100

8. Ann can cut down 90 trees in 3 days, while Mary can cut down 90 trees in 6 days. How many days would it take Ann and Mary together to cut down 90 trees?

- (A) 4      (B) 9      (C) 3      (D) 2      (E) 5

9. How many whole numbers between 20 and 50 (inclusive) have neither 3 nor 5 as a factor?

- (A) 16      (B) 31      (C) 15      (D) 17      (E) 14

10. Five players compete in a chess tournament in which each player plays one game against each of the other players. How many games are played in all?

- (A) 20      (B) 5      (C) 25      (D) 10      (E) 12

11. A square garden has dimensions 10 metres  $\times$  10 metres. In the centre of the garden is a square flower bed with sides of length 2 metres. The centre of the flower bed is exactly in the centre of the garden, and the sides of the flower bed are parallel to the sides of the garden. If walking on this flower bed is forbidden, what is the length in metres of the shortest path from a corner of the garden to the diagonally opposite corner?

- (A)  $10\sqrt{2}$   
(B)  $8\sqrt{13}$   
(C)  $8\sqrt{2} + 4$   
(D)  $2\sqrt{13}$   
(E)  $4\sqrt{13}$

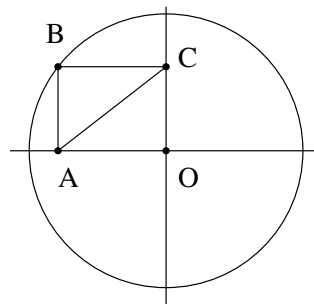
12. What is the remainder when  $2^{10}$  is divided by 5?

- (A) 0      (B) 1      (C) 4      (D) 2      (E) 3

13. A certain street contains 200 houses. If the houses are to be numbered 1 to 200, how many times will the digit 1 appear?

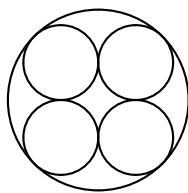
- (A) 138      (B) 119      (C) 139      (D) 120      (E) 140

14. The circle in the following diagram has diameter 10 metres and its centre is the point  $O$ . The figure  $OABC$  is a rectangle and the point  $B$  is on the circle. What is the length (in metres) of the line segment  $[AC]$ ?



- (A) 10      (B) 4      (C) 5      (D) 3      (E) 6

15. In a 100 metre race, each competitor ran at a constant speed. When Aoife crossed the finishing line, Niamh still had 10 metres to run. When Niamh crossed the finishing line, Orla still had 10 metres to run. How far did Orla have to run when Aoife crossed the finishing line?
- (A) 19m      (B) 21m      (C) 20m      (D) 10m      (E) 22m
16. Peter agreed to work for one year for €1,200 and a horse. At the end of seven months, he quit the job and received (as a fair payment) €400 and the horse. What was the value of the horse?
- (A) €400      (B) €300      (C) €720      (D) €840      (E) €640
17. Four people have a total of €100 . Any two of this four have a total of at least €47 . What is the least amount of money that any one of the four could possibly have?
- (A) €25      (B) €20.50      (C) €16.50      (D) €10.73      (E) €22.75
18. Jack is packing the night before his holiday when the power is cut, and he has to finish his packing in the dark (so he can't tell the colour of anything). He knows that in his sock drawer there are 14 socks (and nothing else), of which 6 are black and 8 are blue. He also knows that in the wardrobe there are 9 T-shirts (and nothing else), of which 5 are black and 4 are blue. What is the minimum number of items that he must pack, so that he can be sure of packing one pair of socks and one T-shirt that are all of a matching colour? (Please note that each sock counts as one item!)
- (A) 13      (B) 23      (C) 15      (D) 8      (E) 9
19. The sum of three different prime numbers is 40. What is the result of subtracting the smallest of these three numbers from the largest?
- (A) 25      (B) 29      (C) 20      (D) 28      (E) 17
20. In the following diagram, the radius of the large circle is 1 metre. The small circles are identical in size, they touch the large circle, and some touch each other as shown. What is the radius (in metres) of each of the small circles?



- (A)  $\frac{1}{1+2\sqrt{2}}$       (B)  $\frac{1}{\sqrt{2}}$       (C)  $\frac{1}{2+2\sqrt{2}}$       (D)  $\frac{1}{1+\sqrt{2}}$       (E)  $\frac{1}{2\sqrt{2}}$