



## Problem Solving for Irish Second level Mathematicians

THURSDAY 18TH OCTOBER 2007

### *Senior Level*

Time allowed: **60 minutes**

#### **Rules and Guidelines for Contestants**

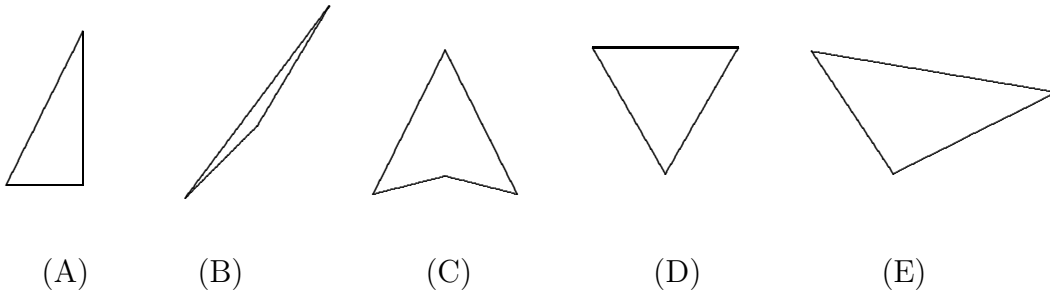
1. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).
2. **Use a pencil to fill out the answer sheet.** If you make a mistake, you can erase the error and correct it.
3. Write your name clearly (in block capitals) in the space provided in the answer sheet.
4. You should have some extra sheets of your own paper (or a refill pad) for rough work while you are doing the questions.
5. When you have decided on your answer for a particular question, carefully mark your choice for that question on the answer sheet.
6. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions.
7. Some of the questions are quite difficult, and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answer to all of the questions.  
The questions at the beginning are easier than those at the end.  
The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions - take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.
8. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

*Good luck and thank you for participating in PRISM.  
We hope you will enjoy the problems!*



*Senior Level 2007*

1. Which of the following is not a triangle?



2. Which one of the following numbers is equal to  $\frac{1}{3} - \frac{1}{12} - \frac{1}{4}$ ?

- (A)  $\frac{1}{7}$       (B) 0      (C)  $\frac{1}{5}$       (D)  $\frac{1}{2}$       (E)  $\frac{2}{3}$

3. Which of the following is the number of centimetres in 10 kilometres?

- (A)  $10^5$       (B) 10,000,000      (C) 1,000,000      (D) 10,000      (E)  $10^3$

4. Which of the following numbers is the largest?

- (A)  $\sqrt{901}$       (B)  $\frac{0.18}{0.006}$       (C)  $2^5$       (D)  $3^3$       (E)  $(\sqrt{5})^4$

5. A square room has diagonal 10m in length. What is the area of the room?

- (A)  $10\text{m}^2$       (B)  $1\text{m}^2$       (C) We need more information.      (D)  $100\text{m}^2$       (E)  $50\text{m}^2$

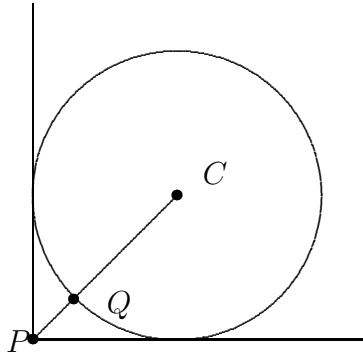
6. What is the value of  $\sqrt{2^4 + \sqrt{3^4}}$ ?

- (A) 5      (B)  $\sqrt{97}$       (C) 25      (D)  $\sqrt{13}$       (E) 7

7. Tap A can fill a tank in 20 minutes, Tap B can fill the tank in 12 minutes, and Tap C can fill the tank in 5 minutes. How long will it take for the three taps together to fill the tank?

- (A) 4 minutes      (B) 3 minutes      (C) 2 minutes      (D) 37 minutes      (E) 5 minutes

8. In the diagram, the horizontal and vertical lines are perpendicular to each other and intersect at  $P$ . The circle just touches both lines, its centre is at  $C$  and its radius is 1. The line  $CP$  intersects the circle at  $Q$  as in the diagram. What is the distance from  $Q$  to  $P$ ?

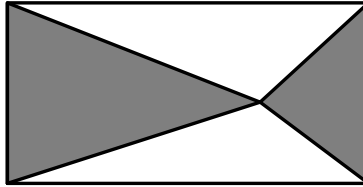


- (A)  $1 - \sqrt{2}$  (B)  $\sqrt{2}$  (C)  $\sqrt{2} - 1$  (D) 1 (E)  $1 + \sqrt{2}$
9. On the first day of a chess tournament with five players, every player plays two games. How many games are played on that day?
- (A) 20 (B) 10 (C) 4 (D) 6 (E) 5
10. In the latest series of *XY-Factor*, contestants competed against each other in solving quadratic equations. In the first programme one quarter of the contestants were eliminated. In the second programme one third of the remaining contestants were eliminated. In the third programme one half of the remaining contestants were eliminated. In the fourth and final programme, five contestants were eliminated leaving one winner of *XY-Factor*. How many contestants took part in the first programme?
- (A) 24 (B) 28 (C) 30 (D) 32 (E) 40
11. The average of a set of ten numbers is 10. If one of the numbers is removed, the average of the remaining 9 numbers is 9. What number was removed?
- (A) 11 (B) 19 (C) 9 (D) 20 (E) 10
12. You are asked to paint each face of a cube with a colour, in such a way that two faces that share a common edge must not have the same colour. What is the minimum number of colours that you will need?
- (A) 2 (B) 6 (C) 4 (D) 5 (E) 3
13. In the sport of prism-ball, five points are awarded for a goal and seven points are awarded for a touchdown. The only ways to score points are through goals and touchdowns. In last week's game, Team A scored a total of 29 points. How many touchdowns did they score?
- (A) 3 (B) 4 (C) 0 (D) 2 (E) 1
14. How many numbers between 100 and 999 have the sum of their three digits equal to 10?
- (A) 54 (B) 100 (C) 63 (D) 46 (E) 55

15. Which of the following is an integer?

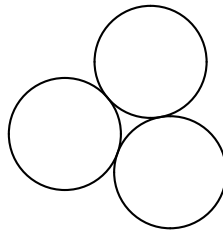
- (A)  $7\pi$  (B)  $\frac{1}{1+\sqrt{2}} + \frac{1}{1-\sqrt{2}}$  (C)  $\sqrt{10^{20}+1}$  (D)  $\frac{2^{50}}{3}$  (E)  $\sqrt{1000}$

16. In the diagram below, the rectangle has area 40. What is the total area of the shaded regions?



- (A) 10 (B) 20 (C) 15 (D) More information is needed (E) 40

17. Three cylindrical barrels each having a diameter of 1 meter and a height of 1 metre are standing side by side on level ground. All touch each other as in the diagram below (which shows the view from above). Find the length in metres of the shortest band that can be tied around the outside of the three barrels.



- (A)  $\frac{3}{2}\pi$  (B)  $3 + \pi$  (C) 6 (D)  $\pi$  (E)  $6 + 2\pi$

18. Conor and Orla are training for a long distance cycling race. Last Saturday morning they started cycling at the same time, Orla from Clifden to Galway and Conor from Galway to Clifden. They both used the same route, and each travelled at a constant speed throughout the journey. Orla reached Galway at 4:00 in the afternoon, and Conor reached Clifden at 2:15 in the afternoon. They met on the road at 12:00. At what time did they start cycling?

- (A) 9:00 (B) 9:30 (C) 8:30 (D) 8:00 (E) More information is needed.

19. Every person in Ballylogic is either a truth-teller or a liar; truth-tellers always tell the truth and liars always lie. One day in Ballylogic, Ann says “Bob is a truth-teller”. Then Bob says “Ann, Cathy and I are . . . all truth-tellers” but at that moment there is a loud noise and you are not sure if he said “Ann, Cathy and I are all truth-tellers” or “Ann, Cathy and I are not all truth-tellers”. Then Cathy (who heard what Bob said) says “Bob said that we are not all truth-tellers”. How many of the three are truth-tellers?

- (A) 1 (B) 2 (C) none (D) 3 (E) More information is needed

20. How many integers  $n$  are there such that  $n + 20$  and  $n - 20$  are both perfect squares? (A *perfect square* is a number that is the square of an integer).

- (A) 4 (B) 2 (C) 1 (D) None (E) Infinitely many